

Algebra I. Homework 2. Due on September 24, 2020.

1. Let $P_n(k)$ be the number of permutations of $\{1, 2, \dots, n\}$ having exactly k fixed points. Show that $\sum_{k=0}^n kP_n(k) = n!$.
2. Give an example of a finite group G and a divisor m of $\#G$ such that G has no subgroup of order m (hint: consider A_4 , $\#G = 12$ and $m = 6$)
3. Show that a finite group G of cardinal $p^\alpha m$ with $(m, p) = 1$ has a subgroup of order p^i for any $i \leq \alpha$. (hint: argue by induction)
4. Let G be a group of order 255. Show that G is not simple (hint: use a p -Sylow of G for some p)