

Algebra I. Homework 10. Due on December, 10, 2020.

1. Prove that S_n is generated by transpositions $(i, i + 1)$.
2. Prove that if $n \geq 3$, then A_n is generated by 3-cycles.
3. Let $L = \text{Frac}\mathbb{C}[x_1, \dots, x_n]$ and $K = L^{S_n}$ where S_n acts by permutations of x_1, \dots, x_n . Let $\sigma_i = \sum_{j_1 < j_2 < \dots < j_i} x_{j_1} x_{j_2} \dots x_{j_i}$ is the i th symmetric function. Prove that $[L : \text{Frac}(\mathbb{C}[\sigma_1, \dots, \sigma_n])] \leq n!$. Then, using the Artin Lemma, prove that $K = \text{Frac}(\mathbb{C}[\sigma_1, \dots, \sigma_n])$.