MATH-GA 2420.006 : Homework 4; due by Monday March 1 morning (before 10am), late submission implies -50% of this homework grade; send the solutions to pirutka@cims.nyu.edu

1. Determine the group of invertible elements $\mathcal{O}_{K,S}^*$ of the ring $\mathcal{O}_{K,S}$ in the following cases: a) $K = \mathbb{Q}, S = \{2, 3\}$; b) $K = \mathbb{Q}(\sqrt{-2}), S = \emptyset$.

2. Let G be an abelian group, $h : G \to R$ a function that satisfies the equality

$$h(P + Q) + h(P - Q) = 2h(P) + 2h(Q), \forall P, Q \in G.$$

The goal is to show that h is a quadratic form : $h(mP) = m^2 h(P) \forall P \in G, m \in \mathbb{Z}$ and $(P, Q) \mapsto \langle P, Q \rangle = h(P + Q) - h(P) - h(Q)$ is a symmetric bilinear form.

   (a) Show that $h(-P) = h(P)$ and that $h(0) = 0$.

   (b) Show that $h(mP) = m^2 h(P) \forall P \in G, m \in \mathbb{Z}$.

   (c) Show that $\langle P + R, Q \rangle = \langle P, Q \rangle + \langle R, Q \rangle$.

   (d) Deduce that h is a quadratic form.