Polygon-ray intersection

Intersecting a ray with a plane

plane: p_0 = point in the plane, n = unit normal

plane equation: $(x-p_0)\cdot n = 0$

ray: $q + v\,t$, q = origin, v = unit direction vector

if $v\cdot n = 0$ (ray parallel to the plane) then

no intersection

$x_{int} = q + v\,t_{int}$, if $t_{int} \geq 0$

$t_{int} = (p_0-q)\cdot n / v\cdot n$

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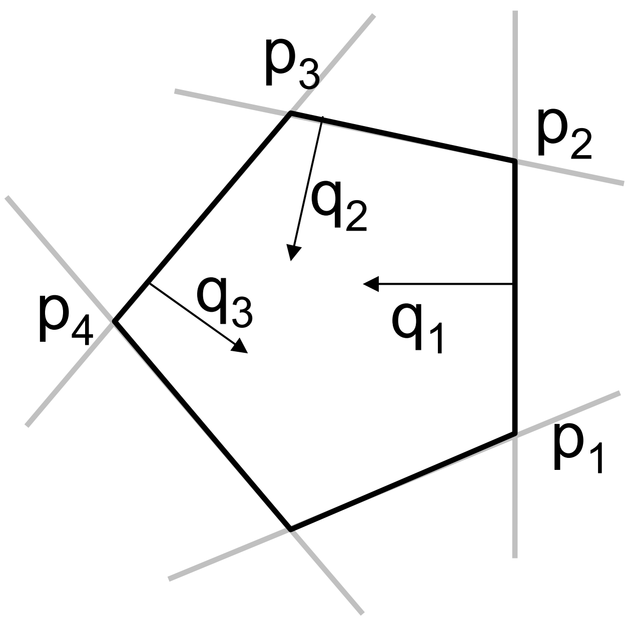
Polygon-ray intersection

Assume convex polygon with N sides.

- Such polygons can be thought of as intersections of half-planes, each bounded by a line extending an edge.



q_i = perpendicular to (p_i, p_{i+1}) in the plane of the polygon; i+1 computed mod N

$q_i = n \times (p_{i+1} - p_i)$

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Polygon-ray intersection

To be inside the polygon, a point x_{inter} should be on the left side of all lines (p_i, p_{i+1})

- Testing for the side of a line (p_i, p_{i+1}):
  - $(x_{inter}-p_i)\cdot q_i \geq 0$ means on the left (cosine of the angle between $x_{inter}-p_i$ and q_i is positive).

Complete algorithm:

- compute x_{inter}
- if for i = 1..N $(x_{inter}-p_i)\cdot q_i \geq 0$, then the point is inside

Far from most efficient, but easy to implement

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Cross-product

Reminder

$c = a \times b$ is a vector perpendicular to both a and b, with length $|a||b| \sin\alpha$, where α is the angle between a and b; the coord. expression is

$[\, a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x]$

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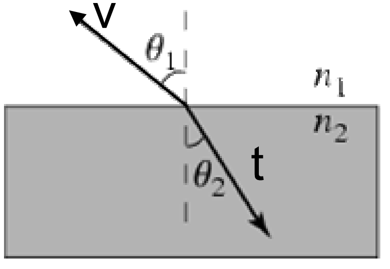
Refracted rays

Snell's refraction law:

$\sin\theta_1 / \sin\theta_2 = n_2/n_1 = n_r$ (relative refraction index)



Unit vector to the eye: $v = v_\perp + v_\|$, $v_\perp = (v\cdot n)n$

Direction of the refracted ray $t = t_\perp + t_\| = |t_\perp| n + (|t_\||/|v_\||) v_\|$

$\sin\theta_1 = v_\|$, $\sin\theta_2 = t_\|$; $t_\| = v_\|/n_r$

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Refracted rays

(continued)

$|t_\perp| = (1-|t_{||}|^2)^{1/2} = (1-|v_{||}|^2 /n_r^2)^{1/2}$

Final formula

$t = |t_\perp| n + (|t_{||}|/|v_{||}|)v_{||} = (1-|v_{||}|^2 /n_r^2)^{1/2} n + v_{||}/n_r$

with $v_{||} = v - v_\perp = v - (v \cdot n)n$

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