Statistical physics of communicating processes

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ideas

idea I

two aspects in solving a distributed problem:
- local steps towards a solution
- backtracking (deadlock escape)

sequential case: can try to always make progress to solution, but NP!

// case: one has to!

idea I - continued

backtrack -> infrastructure (make it a "library")
code easier to prove and understand

universal backtrack strategy

$$p \quad \rightarrow \quad \Gamma \cdot p$$

i.e., add history to a process

results/reversible ccs

1. universal cover ppty:
distributed history characterizes traces up to concurrency

2. weak-bisimulation:
rev(p) + irreversible actions / causal transition system(p) - only irreversible actions observable

3. syntax-independent history construction (eg works for petri nets, pi-calculus)

Jean Krivine

Pawel Sobocinski

2004-2006

Could relax universal cover ppty:

introduce flex-moves (never a choice) not memorized
weak memories: forget synch partner

...

not done anything!

idea II

∞-hesitation, efficiency \
probabilization of rev(p)

exhaustivity \
probabilistic equilibrium

idea III'

What prob structure?
borrow from stat phys

distributed CT Metropolis

build a potential energy function
drive kinetics (Newtonian style, stochastic version)

build a **causal/concurrent**, and **convergent** potential energy on the state space of reversible CCS

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the reversible CCS transition system

reversible communicating processes

memory

n-fork

$$\Gamma \cdot (p_1, \ldots, p_n) \rightarrow^f \Gamma 1 \cdot p_1, \ldots, \Gamma n \cdot p_n$$

synch on a_1, ..., a_m

$$\Gamma_1 \cdot (a_1 p_1 + q_1), \ldots, \Gamma_m \cdot (a_m p_m + q_m) \rightarrow^s_{\vec{a}}$$
$$\Gamma_1(\vec{\Gamma}, a_1, q_1) \cdot p_1, \ldots, \Gamma_m(\vec{\Gamma}, a_m, q_m) \cdot p_m$$

with a unique naming scheme and enough info to reverse uniquely symmetric TS (so strongly connected)

"simplicity" of TS: at most one jump

slight pb with sums between X and Y

near acyclic

countable state space (recursion)

potential/rate ratio constraint

CTMC

$$q(y,x)/q(x,y) = p(y)/p(x) = e^{-(V(y)-V(x))}$$

$$\sum_X e^{-V(x)} < \infty$$

that is convergence by def!

NB: lower energy/higher probability

explosive growths

		event horizon	nb of com
$q \to^f$	$0 \cdot p(a), 1 \cdot p(\bar{a})$	$1, 1$	1
$\to^{fs}$	$0a0 \cdot p(a), 0a1 \cdot p(a), 1\bar{a}0 \cdot p(\bar{a}), 1\bar{a}1 \cdot p(\bar{a})$	$2, 2$	2
$=$	$0a0 \cdot a(p(a), p(a)), 0a1 \cdot a(p(a), p(a)),$		
	$1\bar{a}0 \cdot \bar{a}(p(\bar{a}), p(\bar{a})), 1\bar{a}1 \cdot \bar{a}(p(\bar{a}), p(\bar{a}))$		
$\to^{fs}$	$0a0a0 \cdot p(a), 0a0a1 \cdot p(a), 0a1a0 \cdot p(a), 0a1a1 \cdot p(a),$	$4, 4$	$4!$
	$1\bar{a}0\bar{a}0 \cdot p(\bar{a}), 1\bar{a}0\bar{a}1 \cdot p(\bar{a}), 1\bar{a}1\bar{a}0 \cdot p(\bar{a}), 1\bar{a}1\bar{a}1 \cdot p(\bar{a})$		
$\ldots$			
$\to^{fs}$	$\prod_{w \in 2^k} 0w(a) \cdot p(a), \prod_{w \in 2^k} 1w(\bar{a}) \cdot p(\bar{a})$	$2^k, 2^k$	$2^k!$

is there a concurrent potential that controls the above?

upper bound on the number of such (entropy)

lower bound on energy of a deep state

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Construction of a potential

total stack size potential

$$V_1(p_1, \ldots, p_n) = V_1(p_1) + \ldots + V_1(p_n)$$

$$V_1(\Gamma \cdot p) = V_1(\Gamma i) = V_1(\Gamma)$$

$$V_1(\Gamma(\vec{\Gamma}, a, q)) = V_1(\Gamma) + \epsilon_{\vec{a}}$$

$$\vec{\epsilon} \cdot \tilde{\Gamma}(p)$$

V_1 energy balance for forks and synchs

$$\Delta V_1 = (n-1)V_1(\Gamma) \qquad n\text{-ary fork with memory } \Gamma$$
$$\Delta V_1 = m\epsilon_{\vec{a}} \qquad \text{synch on } \vec{a}$$

realize the ratio constraint as:

$$k_f^- = 1 \qquad k_{\vec{a}}^- = 1$$
$$k_f^+ = e^{-(n-1)V_1(\Gamma)} \qquad k_{\vec{a}}^+ = e^{-m\epsilon_{\vec{a}}}$$

total synch potential

Given a path γ from $\varnothing \cdot p_0$ to p:

$$V_0(p) \quad = \quad \sum_{\vec{a} \in A^\star} \sum_{x \xrightarrow[\vec{a}]{s} y \in \gamma} (-1)^{v(s)} \epsilon_{\vec{a}}$$

ratio constraint:

$$k_f^- = k_f^+ \qquad\qquad k_{\vec{a}}^- / k_{\vec{a}}^+ = \exp(\epsilon_{\vec{a}})$$

V_1 is truly concurrent, i.e. sensitive to sequential expansion

V_0 < or equal to V_1

potentially more divergent

No matter how costly a synch, V_0 diverges

What about V_1 ?

upper bound on the number of such (entropy)

Lemma 5 *For large ns,* $\log|T(n)| \leq \beta_+\alpha^2 O(n\log n)$

lower bound on energy of a deep state

Lemma 4 *Suppose* $\beta_- > 1$, $\epsilon_m > 0$, $p \in \Sigma_n(p_0)$:

$$\frac{\epsilon_m}{\log 4 + \log(\beta_+ + 1)} \cdot n\log n \leq V_1(p)$$

Sufficient condition for equilibrium

Proposition 1 *Suppose* $1 < \beta_-$, *and* $\beta_+ \alpha^2 \log(4(\beta_+ + 1)) < \epsilon_m$, *then:*

$$Z(p_0) \quad := \quad \sum_{p \in \Omega(p_0)} e^{-V_1(p)} \quad < \quad +\infty$$

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epilogue

Simulated annealing with "local" temperatures

$$k_f^- = 1$$
$$k_f^+ = e^{-(n-1)V_1(\Gamma)}$$

the bounds are rough

Control growth rate?

What with irreversible actions?

other potentials?

Work with general steady states?

reactive modules? something else than CCS

What kind of problem?

obvious connexion with rewriting theory

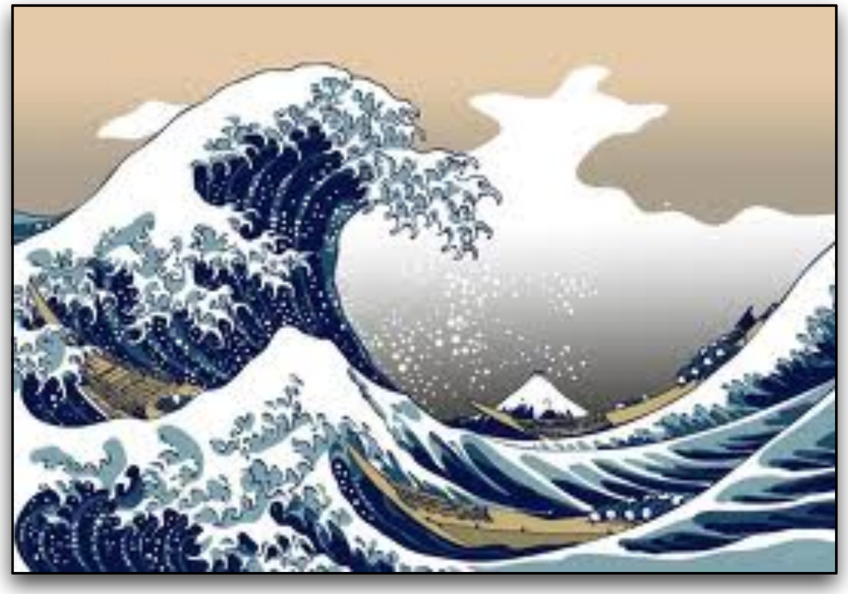
idea III:
energy-based programming/distributed Metropolis
code = statics/potential + transition/moves + compatible kinetics

energy as syntax

self-organised energy-based dynamics

... stochastic machine learning

$$\text{argmax}\,\vec{\epsilon}.\ \sum_{p\in\partial X}\pi(\vec{\epsilon},p)=\int \mathbf{1}_{\partial X}\,d\pi$$



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