Problem 1:

1) \[ w_p = \left( \frac{C m_p - A}{D} \right) V^{-1} m + \left( \frac{B - A m_p}{D} \right) V^{-1} 1 \]

where
\[ A = m^T V^{-1} 1, \quad B = m^T V^{-1} M, \quad C = 1^T V^{-1} 1, \quad D = B C - A^2 \]

For the given assets we have:

\[ A = -\frac{1}{2} \]
\[ B = 7.8 \]
\[ C = 0.4167 \]
\[ D = 3 \]

\[ w_p = \begin{bmatrix} 1.19 \\ 0.07 \\ -0.26 \end{bmatrix}. \]

b) \[ w_p^* \] is found by setting \( m_p^* = \frac{A}{C} = -1.2 \)

this gives \[ \theta_p^* = \frac{1}{C} = 2.4 \]

\[ w_p^* = \begin{bmatrix} 1.4 \\ 0 \\ -0.4 \end{bmatrix} \]

c) Yes, since \( m_p > \frac{A}{C} \).

d) \( w_q \) is found by setting \( m_q = \frac{A}{C} - \frac{D/C^2}{(m_p - A/C)} \)

\[ w_q = \begin{bmatrix} 3.6857 \\ -0.7669 \\ -1.9238 \end{bmatrix} \]
e) \( m_q = -14.9148 \), \( s_q^2 = 28.5225 \)

f) \( q \) is not efficient since \( m_q < \frac{A}{c} \)

We remark that while some returns above are negative, the theory still holds and could still describe desirable investments depending on the numeraire considered.
Problem 3:

1) By the CAPM, the risk premium of an asset in the market place is given by:

\[ M_A - r = \beta (M_m - r) \]

where:
- \( M_A \): asset expected return
- \( M_m \): market expected return
- \( r \): risk-free interest rate
- \( \beta = \frac{\text{covar}[P_A, P_m]}{\sigma_m^2} \)

For the stock this requires the return with:

\[ \beta = 1.50 \times 2 \quad \text{and} \quad r = 0.05 \quad \text{so} \quad M_m = ? \]

\[ M_A = 0.05 + 1.50 \times 2 (M_m - 0.05) \]

For a market return of \(10\%\) we have:

\[ M_A = 0.125 \] = 12.5 \% \]

2) Let \( P_m \) be the return of the market portfolio (in other words \( P_m = \sum_i w_i P_i \), where \( w_i = \frac{\text{market cap of asset}}{\text{total market cap}} \)).

Vow from Markowitz's Theory, the optimal portfolio consisting of the N risky assets plus a risk-free asset is a linear combination of \( M \) and a risk-free asset \( B \).

\[ P_p = (1-\alpha) P_0 + \alpha P_m \]

\[ \text{Var}[P_p] = \alpha^2 \text{Var}[P_m] = \alpha^2 \sigma_m^2 \]

\[ \mu_{P_p} = \alpha \mu_m \]

Thus \( \sigma_{P_p} = \sigma_m \alpha \)

Thus \( \alpha = \frac{\sigma_{P_p}}{\sigma_m} = \frac{1}{2} \)

When \( \alpha = \frac{1}{2} \quad \text{and} \quad \mu_{P_p} = \frac{1}{2} B + \frac{1}{2} M \)

Thus \( \frac{1}{2} \mu_{P_p} = \frac{1}{2} \frac{1}{2} B + \frac{1}{2} M \)

\[ \frac{1}{2} \mu_{P_p} = \frac{1}{2} \text{market cap stocks} - \frac{1}{2} (1 + 1) \mu_m \]

\[ \mu_{P_p} = 2.5 \times 10^{-4} \]
\[ M_A = r + \beta (\mu_m - r) \]
\[ = 0.05 + 0.3489 (0.10 - 0.05) \]
\[ = 0.0624 = 6.24\% \]

**Price today = \( \frac{1}{(1 + M_A)} \) (Price in one year)**

\[ = \frac{1}{1.0624} \approx 0.94127 \] (bid price to ensure proper return)