1) **OUTLINE**

(a) NP-Hard to determine if a graph has an embedding with Resolution $\frac{\pi}{d}$.

(b) Resolution of outerplanar graphs.

(c) Upper bound on the resolution of a Random graph.

2) **Review**

(a) Resolution: Minimum angle between edges incident on the same vertex in a straight line embedding of the graph.
2) **NP-HARDNESS**

(a) What is **NP-Complete Problem?**
**NP-Hard Problem?**

(b) How to Prove a Problem is NP-Hard/Complete?

(c) Example of NP-Complete Problem: SAT, 3 SAT

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**Theorem**

Given a graph \( G_n \) of max degree 4, the decision problem of whether or not \( G_n \) can be embedded in the plane with resolution \( \Pi^2/2 \) is **NP-HARD**.

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**Proof:**

Reduction from 3-SAT.
The skeleton.

The Tower

1) Place a tower on each variable in the skeleton.
Clause $C_i$ contains $x_i$

Note:

(i) Each clause has at most 3 connections.

(ii) In how many ways can the connections be embedded?
Angular Resolution of Outer planar Graphs

Theorem

Every outer planar graph with max degree has a planar straight line embedding where each face is an isosceles triangle with base angle $\alpha = \frac{\pi}{2(d+2)}$

Lemma:

$|PD| = l = s \sum_{i=0}^{\infty} y_i$

$\alpha = \frac{\pi}{2(t+2)}$ where $t \geq 2$

Then:

$(t-1)\alpha + B < \frac{\pi}{2}$
we will prove a stronger statement:

Additional Property:
Embedding of $G$ is contained inside triangle $ABP$.

Also: $(\text{degree}(A) - 1)x + \angle PAC < \frac{\pi}{2}$
Resolution of a Random Graph:

A "Handwaving" Proof!!

**Theorem:**
"Many" graphs with max degree \( d \) have resolution at most
\[
O\left(\frac{\log d}{d^2}\right)
\]

**Proof:**

(i) For simplicity, consider directed graphs with out-degree \( d \).

(ii) Consider only angles formed by outgoing edges.

(iii) Number of \( N \)-node out-degree \( d \) graphs:

\[
\binom{N}{d}^{N-1}
\]
Lemma

Given any set of N points in the plane, it is possible to find Concentric Circles with radii \( r_1 \) and \( r_2 \) such that at least \( \frac{N}{15} \) points are inside or on the boundary of the inner circle and at least \( \frac{N}{15} \) are outside or on the boundary of the outer circle where \( r_2 \geq 9r_1 \).