Computational Geometry
Final Exam, May 8, 2002

1. Prove that in the Algebraic Decision Tree model to decide whether n given numbers are all *distinct* requires at least constant times $n \log n$ queries (not necessarily comparisons!).

2. Let $p_1, p_2, \ldots, p_n$ be the vertices of a convex polygon listed in clockwise order. Design a $O(n)$-time algorithm for finding the shortest distance between two vertices.

3. Given a set $P = \{p_1, p_2, \ldots, p_n\}$ of n points in the plane, for any i, let $F(p_i)$ denote the set of those point in the plane from which p_i is *at least* as far as any other point p_j $(j \neq i)$. (The "cells" $F(p_i)$, $1 \leq i \leq j$ are usually said to form the so-called *farthest-point Voronoi diagram* of P.)

Prove that each cell $F(p_i)$ is convex.

4. Recall that the *size* of a BSP tree (Binary Space Partition tree) for a set of segments is equal to the total number of segment fragments generated by the splitting lines. An *autopartition* is a BSP-tree in which every splitting line contains one of the segments.

Give an example of a set of n non-intersecting line segments in the plane, for which a BSP tree of size n exists, but any autopartition has size at least $\lfloor 4n/3 \rfloor$.

5. Let S be a set of n circles in the plane.

Outline an algorithm to compute all intersections between the circles in $O((n+k)\log n)$ time, where k is the output size, i.e., the number of intersection points.

6. Consider a polygon P of n vertices in the plane, which is triangulated by some of its internal diagonals.

Design a $O(n)$-time algorithm for coloring the vertices of P by 3 colors so that no 2 vertices that are connected by an edge or a diagonal get the same color.

Please explain all of your answers! Good luck! - J.P.