Combinatorics

Final Test

1. Prove the identity

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}.$$

2. 10 people sit at a round table. In how many different ways can we pick 3 of them so that no two sit next to each other?

3. Let T be a tree with x leaves and y vertices of degree at least 3. Show that $y \leq x - 2$.

4. Prove that if in a bipartite graph every vertex has degree at most k, then by adding vertices and edges we can obtain another bipartite graph, in which every vertex has degree precisely k.

5. What is the maximum number of edges that a graph of n vertices can have if it does not contain any triangle whose one vertex is connected to an extra (fourth) vertex?

Please explain all of your answers! Good luck! - J.P.