Two places at once
(A remembrance of Paul Erdős)

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Anyone entering the faded old building of the Mathematical Institute of the Hungarian Academy of Sciences in the past twenty years could immediately tell whether Paul Erdős was in town. If he was, the large black door of the director's office was wide open, and the place was bustling with people. A secretary would be typing Erdős's latest manuscript and complaining about his scratchy, childish handwriting. Another secretary might be on the phone, trying to make airline reservations for Erdős's forthcoming lecture tour through several continents. The librarians would be busy locating articles he requested, and running upstairs to xerox them. There would be a steady stream of friends and colleagues, here to take up the thread of conversation or work left off the day – or sometimes several months – before.

Meanwhile, Erdős would be sitting in the director's velvet-upholstered armchair, in the middle of a phone call. He was almost always in the middle a phone call. Or dialing. Or cursing out the telephone company because all the lines were busy. (Oddly enough, most of his telephone calls were quite short. After asking a couple of quick questions, he usually sketched his latest ideas related to a mathematical problem. Finally, he would list all the places and telephone numbers where he could be reached during the rest of the day.)

It was only natural that he should occupy the director's office. Here he was closer to the secretaries, the armchairs were more comfortable and, what's more, from this office one could make direct international calls. Every director of the Mathematical Institute since its founding in 1950 has been a good friend of Erdős. They were more than happy to let him use their office. Paul Erdős was a "monarch", after all. He was one of the brightest mathematical luminaries of our century. Though it is too early to judge the true dimensions of his influence on mathematics and mathematical life, it is clearly enormous.

1 Simultaneous games

Most of us, ordinary mortals, do not know which way to turn when several people want to see us at the same time. But Erdős was at home in situations like this. He was very proud of his ability to do several things, simultaneously. He could chat with a guest about politics, analyze two different mathematical problems with two others, and read a book – all at once. He would embarrass his hosts when on top of these activities he'd also contend with dinner. The guests would all be seated, waiting for the first course, when all of a sudden Paul would jump up and rush to the telephone to call someone he had been working with earlier that day. He would quickly explain a fresh idea that struck him as relevant to a problem, and, possibly, the key to its solution. By acting this way, he would set a difficult task for his host; after dinner he'd have to convince his spouse that Erdős was not a rude man, he did not want to insult anyone. In fact, when any of his friends was in serious trouble, he called immediately and tried to be of help. Nevertheless, in his universe mathematics stood so high above the ordinary aspects of life that he sometimes could not resist the temptation to drop everything else.

He dazzled his acquaintances by his knowledge of everyone's telephone number. Although he published an enormous number of papers, when someone asked about any of his theorems, he always remembered the title of the paper, the name of the journal where it appeared, and the year of publication. He even knew the exact page numbers. But he was gifted with total recall of more than just numbers. Sometime towards the end of the fifties, in the guesthouse of the Hungarian Academy at Mátraháza, my father introduced him to a colleague of his, the historian Lajos Elekes. Paul, who was interested in practically everything, immediately engaged him in a conversation. It was his habit to ask people: "What's your profession?". As soon as he found out that Elekes was writing a book about the fifteenth century Hungarian general and governor, János Hunyadi, he proceeded to inquire about the causes of the catastrophic defeat of the Hungarian army by the Turks in the battle of Varna, in 1444. The conversation soon broke up, perhaps because it was lunchtime, and continued a year later at the same place. When Paul noticed Elekes, he recognized him right away and shouted from afar: "So why did Hunyadi lose at Varna?"

Roughly fifteen years later, at the same place, I witnessed Paul approaching the famous Hungarian poet, Ferenc Juhász. "What's your profession," he asked, as usual. Everybody around felt somewhat embarrassed, because Juhász then was at the peak of his career. The whole country knew his face, especially his characteristic moustache. He often appeared on TV, the literary magazines and supplements were full of his poetry, and he published one volume after another. Wherever he went, he was instantly recognized. He obviously enjoyed his being a celebrity and was not used to questions like that. "I am a poet," he declared proudly. Paul innocently asked: "And how do you make a living?" Paul spent relatively little time in Hungary, he almost never watched TV, and he did not regularly follow the Hungarian journals. So he could hardly have heard of Juhász before. He certainly did not want to offend him, though. In fact, I doubt that he ever wanted to hurt anyone's feelings.

It is true, however, that he never paid much attention to titles, ranks, and decorations. If he was interested in a question, he was eager to discuss it with anyone, be he an Einstein or a cabdriver. He was much less prejudiced than anyone else I have ever met. Although he received (and accepted) some thirty honorary degrees and memberships in distinguished academies, he attributed them partly to his old age. He always said that he would be happy to trade all of his titles for a beautiful theorem. He meant it, too.

2 Child prodigy

Pál Erdős was born in Budapest on March 26, 1913. While his mother was in the hospital giving birth, her three- and five-year-old daughters fell ill with scarlet fever and died. Paul grew up as an overprotected only child. Fearing of contagious diseases, his parents kept him away from public schools, and he completed most of his studies as a private student. Both his parents were high school mathematics teachers, so they instructed Paul themselves. But they also hired tutors and a German governess. Paul's father fell into Russian captivity during WWI, and returned to Hungary only in 1920. This was the first time he could talk to his son. Soon after that, he started to teach Paul English and introduced him to the realm of prime numbers.

In a recent lecture at Eötvös University, Budapest, Paul described the child he once was as a "bit of a prodigy." When he was four, he discovered negative numbers, and he was able to multiply four-digit integers in his head. From the age of thirteen he was a regular problem solver of KöMaL, a first-rate Hungarian mathematics journal for high school students. At the end of every year, his picture appeared among the photos of the most successful students. In 1930, he was admitted to the university of Budapest, and in the same year (at the age of seventeen!) he made his first important mathematical discovery: he found an elementary proof to the famous theorem of Chebyshev, according to which between any positive integer and its double there is a prime number. He liked to quote Nathan Fine's words:

Chebyshev said and I say it again:
There is always a prime between n and $2n$.

In his doctoral dissertation, written under the supervision of Leopold Fejér, he proved some far-reaching generalizations of this theorem. The year he defended his thesis, he was only twenty one. During his student years, he met Tibor Gallai, György Szekeres, and Paul Turán, who became lifelong friends and collaborators. They often attended Dénes Kőnig's lectures on graph theory at Budapest's Technical University. Answering a question raised in one of these lectures, the eighteen-year-old Erdős found an exciting extension of a theorem of Menger to infinite graphs. (His proof was included in Kőnig's classical monograph published in 1936.) One year later, Erdős and Szekeres made a brilliant discovery that led to the publication of their first joint paper. It is rightly regarded now as the starting point of Ramsey theory and combinatorial geometry.

From 1934 to 1938, Paul worked in Manchester and spent only his vacations in Hungary.

He often recalled that before he left for England, he had never had to butter his bread himself. This turned out to be a fairly easy skill to master, but he never learned how to prepare food for himself. Once I spent a few days with Paul in his apartment in Budapest. When I entered the kitchen in the evening, I was met with a horrible sight. The floor was covered by pools of blood-like red liquid. The trail led to the refrigerator. I opened the door, and to my great surprise saw a carton of tomato juice on its side with a gaping hole. Paul must have felt thirsty and after some reflection, decided to get the juice out of the carton by stabbing it with a big knife.

The many mathematics contests for elementary and high school students, the problem sessions of KöMaL, and the special mathematics programs in a network of selected schools in Hungary have always brought to the surface some extraordinarily gifted children. Paul never missed an opportunity to talk to them. He tried to accept all invitations, whether they came from an elementary school in a remote provincial town, a topnotch Budapest *Gymnasium*, or Trinity College, Cambridge. Some of the most talented children he discovered were Attila Máté, Lajos Pósa, and Imre Ruzsa. Pósa was a real child prodigy. He was twelve when the outstanding Hungarian logician and pedagogue, Rózsa Péter, introduced him to Paul, and one year later Erdős and Pósa were already writing a joint paper in graph theory.

Paul referred to children as "epsilons". He loved to hold babies in his arms. While doing this, he usually asked: "Isn't it remarkable how calm this epsilon is?" (The parents, especially if they did not know him, were less calm.) At airports, in restaurants, on the streets, he would accost people with small children. "How old is this epsilon? Very cute! Tell me, is it a boss or a slave?" (At this point, someone had to explain to the astonished parents that the question referred to whether the child was female or male, respectively.) If the child was not so young, he or she could hardly get away without seeing the following performance. Paul would pull a coin from his pocket and place it on the back of his hand. Then he would suddenly snatch his hand away, letting the coin fall, but catching it with the same hand before it hit the ground. Most kids would just stand there gaping. Either because they appreciated the difficulty of the trick, or because they had no idea what this strange man was doing. But they rarely cried. When the conversation shifted to epsilons, Paul liked to recall the following episode. His mother asked a lady how many children she had. "Seven," she answered. Aunty Annus (this is what I called Paul's mother), who could never get over the tragic loss of her daughters, nodded appreciatively: "Then you are a rich woman."

3 Fight for freedom

It is very likely that in the past twenty years Paul Erdős became the best known mathematician of the world. His unique fame and popularity is explained mainly by his unparalleled scientific achievements and also by the fact that he was in contact with almost all the leading mathematicians of his time. He published joint papers with 458 people, but the number of those whose career was profoundly influenced by his innumerable questions, problems – and over 1500 articles – is much higher. Most people are fascinated by records, and it is impossible to compete with the above numbers. They are staggering.

However, one has to be careful with numbers. They may lead us to believe that we are closer to the understanding of an extraordinary phenomenon than we really are. A movie by Werner Herzog ends with the dissection of the body of the main character, Kaspar Hauser, who has had a peculiar life and an original way of thinking. The doctors and scholars are relieved to find that his liver was oversized and his brain also had some special features. They have no doubt that these irregularities account for Kaspar Hauser's curious personality. Allegedly, Einstein's brain was also subject to thorough analysis in an attempt to unlock the ultimate secret of his genius.

In most of Erdős's obituaries, it was emphasized that he led an unassuming life. He owned almost nothing, not even books. He traveled around the world with two half-empty suitcases, containing only his laundry, his toilet bag, and his pocket radio. After the death of his mother in 1971, which he never quite got over, he traveled mostly by himself. He found it hard to stay in the same town for longer than two weeks. As mathematicians like to say, Paul was "everywhere dense" on the earth. If you want to see him, just stay in one place and wait – he will soon appear... He remained single all his life and he never took a job that required serious commitment. If his work or scientific interest drew him to another place, he did not hesitate for a moment. He was ready on a few hours notice to fly to, say, Singapore with only a hundred dollars in his pocket. Wherever there were active mathematicians, he was most welcome, so he did not have to worry about accommodations and money. This lifestyle, free of conventional restrictions, must also have contributed to Erdős's colossal scientific success.

It is not an accident that Paul developed such a lifestyle. Fate and history were not so gracious to him as, say, to Gauss. They did not lay at his feet the prospect of a peaceful and comfortable middle class life. In 1919, during the short-lived Hungarian Soviet Republic, Paul's mother was headmistress of a high school. Because of this, she was later banned from teaching for the rest of her life. This "bit of a prodigy", who grew up in a strongly anti-Communist and anti-semitic political environment, felt from the very beginning that sooner or later there would be no place for him in his own country. When it became clear that Germany's appetite for territories was not satisfied by the Anschluss, and that it would soon attack Czechoslovakia, Paul said goodbye to his parents and left Hungary. When he returned ten years later, none of his close relatives – save his mother and one of his aunts – were alive.

Like his parents and most of his lifelong friends, Paul had always been a leftist at heart. But he was never blind to the mistakes and crimes committed by the Communist regimes.

As soon as the first show trials got underway in Hungary, Paul packed his suitcases again, said goodbye to his mother, and left. By the time the first victim of these trials, László Rajk, was sentenced to death, Paul was back in the United States – but with no steady income. He was already a famous mathematician, yet he was not exactly overwhelmed with offers. He was not interested in an academic position with regular teaching duties, but he would not have rejected a serious research fellowship. He was also more and more concerned about the chilling effects of the McCarthy era. He sadly recounted the time when he wanted to telephone his mother from a friend's house to congratulate her on her 70th birthday. His host, however, did not allow him to make the call, because he did not want to be suspected of associating with Communists. This fear was not unfounded. Soon afterward it turned out that the FBI had put together a sizeable file on Paul.

They kept records of his vast correspondence, and held it against him that he was in contact with colleagues from Communist countries, especially Hungary and China (e.g., collaborated with Lo Keng Hua). In 1954, Paul was invited to speak at the International Congress of Mathematicians in Amsterdam. He needed a re-entry visa to the United States. The Immigration Service sent an officer to the Mathematics Department of the University of Notre Dame, where Erdős was working at the time. "What is your opinion about Marx?" – the officer asked him during the course of an interview. "I do not feel competent to judge Marx," Paul answered "because I have read only the *Communist Manifesto* by him. But I do believe he was a great philosopher." Then he was asked whether he would visit Hungary if the Hungarian authorities guaranteed that he could leave the country whenever he wanted to. "Of course I would!" he replied. "My mother lives there as well as some of my best friends." His visa application was turned down, but Paul decided to participate in the congress, anyway. He packed again and left the United States. He was not allowed to return for nine years. His American friends had begged him to stay, wait a year and submit a new petition. Before he left, he spent his last night at the home of Harold N. Shapiro, who fiercely criticized his decision: "I should knock you on the head and tie you up to stop you from leaving!" Paul remained firm. He sadly replied: "O.K., then tie me up!" His case was not reconsidered until 1963, two years after Kennedy became president.

From August 1955 on, Paul spent a couple of months a year in Hungary. In 1973, again for political reasons, this pattern was broken for more than two years. The Bolyai Mathematical Society celebrated Erdős's sixtieth birthday by organizing a big conference at Keszthely with hundreds of participants. The Hungarian authorities denied an entry permit to the Israeli guests. Paul was furious. He called a high-ranking party official and lodged a vigorous protest, but to no avail. He vowed not to return to Hungary for a long time. Indeed, he did not return until 1976, and then only to visit one of his best friends, Paul Turán, who was dying.

Thus, one cannot explain Erdős's vagabond life style by simply noting his restless nature. He chose to be free, and for that he had to make serious sacrifices. He did not let anyone restrict his free movement and work, not Hitler, not Joe (after Joseph Stalin – this is how he referred to Russia and to the Communist powers), and not Sam (the United States), either. He did not believe that the glory of the superpowers would last forever, but Joe's sudden collapse surprised him. In the spring of 1989, I bet him 5000 forints that the Soviet Union would disintegrate within a year. I lost, but later Paul admitted that I had won a "moral victory". He added, with a little concern in his voice:

Once Sam and Joe went up the hill
To fetch a pail of water.
And Joe fell down and broke his crown
And Sam came tumbling after.

4 Golden age

In 1956, Paul Erdős was elected a member of the Hungarian Academy of Sciences. He did not care about ranks or titles, but I am sure that he took this membership a little more seriously. He often flew back from overseas for a few days to participate in the general assembly and to cast his vote for new members. When one of his colleagues was elected to be a corresponding member, he typically congratulated him, as follows: "I am glad that you became a demigod!"

Several times a year Paul and his mother spent a couple of weeks in the guesthouse of the Academy at Mátraháza. Most of the guests were scholars, but there were also a few writers. Paul usually tried to arrange his program so that he and his mother would come when the Turáns and my parents were also there. On such occasions, they laid a big table for the ten of us in the dining room of the guesthouse. Most other tables were small, and they were occupied by elderly couples who rarely spoke to each other. Sometimes I caught their envious glances, because our table was so much livelier. The three Pauls, Erdős (P.E.), Turán (P.T.), and Pach (P.P.), were a fount of stories, jokes and ideas. (They used to call each other by their initials, like newspapermen, who sign their articles that way.) It lent a special flavor to the conversation that P.E. often talked to his mother in English. Aunty Annus started to accompany her son on his trips when she was eighty four, and she decided to learn English. I will never forget how she would turn to Paul and ask: "Palkó, how do you call the fruit 'szilva' in English?" "Plimm, mother, plimm!" was his answer. It is quite remarkable that, although Erdős started to study English at the age of eight and spent most of his life in English-speaking countries, he had a very heavy accent. Recently, a documentary film was made about him, including interviews with several other Hungarian mathematicians. Paul was the only one whose English had to be subtitled. He did not have a good ear for music, and he did not particularly like it either. Turán loved to listen to music while working, and he knew the classical repertoire very well. When Erdős entered

Turán's study, he would stop and ask: "What is this noise?"

It was among Erdős's and Turán's favorite diversions to rewrite well-known lines of Hungarian poetry. The central theme of their "compositions" was old age and senility, the two things that terrified them most. For instance, Erdős paraphrased two famous lines by Sándor Petőfi as follows.

One thought disturbs me, that I may decease
In slowly progressing Alzheimer's disease.

Erdős quoted these "masterpieces" often and with pride, their questionable literary merit nothwithstanding. Almost twenty years after Turán's death, Erdős no longer remembered exactly which line was written by him and which by Turán. Both of them loved the poems of Endre Ady and the "Westerners", a group of progressive Hungarian poets and writers who published their work in the periodical "Nyugat" (West). Whenever Ady's name was brought up in a conversation, Erdős remarked: "He was a great poet, but a trivial person!" (In Erdős-ese, 'trivial' is 'mean'.) Paul never forgave Ady for breaking up with his long-time mistress by writing a devastating and humiliating farewell poem to her.

"Have a game of ping-pong with me," said Turán "and I tell who you are." Indeed, if one pays attention, one can often sense how inventive, how generous one's partner is, how he can deal with failure or bad luck, how he can hold up under pressure, etc. As soon as Erdős took his stand behind the ping-pong table, it was clear that he was an amateur and could not be a serious opponent. He even held the bat in a strange way, as if he did not want to soil his hand with it. His serves were totally ridiculous; it was not hard to smash them. But then came the surprise: Paul easily repelled the attack! His reflexes were fantastic. One could not help thinking that in his nervous system impulses somehow traveled a lot faster. Turán had much better technique, he affected a Chinese style with spectacular motions. He was very proud of having won a table-tennis championship on board the Queen Mary. Erdős and Turán always put up a good fight. Turán moved around like a pro, while Erdős remained still and blocked the shots. When he missed a ball, he shouted out: "Fascism!" Both of them liked to play with us, kids, and they almost always won. If any of us managed to place a good shot, Erdős exclaimed with a smile of recognition: "Look! The epsilon does not surrender! He's fighting on..." He defeated me even in 1984, in Calgary, in the basement of Eric Milner's house, although by that time his eyesight had weakened considerably. He did not see the high balls at all.

At Mátraháza, the three Pauls (Erdős, Turán, and Pach) took long excursions in the surrounding hills. Erdős went along mainly for the company. And, of course, it never hurt to get some exercise. Turán enjoyed thick, overgrown forests, and nature in general. He liked to explore new pathways – just as in his mathematics – and we were therefore often late for lunch. One of his favorite slogans was "Don't stray from the unbeaten path!" All three had the same irresistible adolescent impulse to climb each and every peak they saw. Not far from the guesthouse at Mátraháza, there is a fairly tall, steel observation tower.

One has to climb up three steep ladders to get to the top. Quite a few rungs are missing, and the whole structure shows the ravages of time. Until very recently, Erdős, whenever he visited the guesthouse, insisted on clambering up this tower, defying the warnings of his anxious friends. I will never forget Turán' own determined attempt to get to the top in the summer of 1975, when he was already fast losing his strength.

I also vividly remember Erdős and Turán working together in Mátraháza. They were often joined by Vera Turán-Sós (my aunt) and Alfréd Rényi, who was known to everybody as "Buba". Turán often chided Erdős for talking "nonsense" and asked him to rephrase his ideas more precisely. Erdős had an incredibly quick mind, and he wanted to explain everything at the same rate of speed, occasionally skipping and jumping. (Only much later, when I was a senior at Eötvös University and I started to collaborate with Erdős myself, did I fully understand why Turán had complained so vehemently. Sometimes it took me a day to work out the details of a proof that had been outlined by Erdős in five minutes. And ten years earlier he must have been even faster!) The misunderstanding was often sorted out by my aunt, who was better at reconstructing an argument from fragments. Rényi's dashing and witty comments, which even an epsilon could appreciate, made it even more fun to watch them. When the grownups went for a walk, or played ping-pong, or were having coffee, and the scene became quiet, I sneaked over to the table to catch a glance at "higher mathematics", i.e., at the notes scattered on the table. I was astonished when I first saw the end-product of their work: strange letters, numbers, signs, arrows, scribble-scrabble. Yet I felt that I, too, would like to be a scientist. I had no doubt that the Laws of the Universe were written in this mysterious language. Otherwise, how could mathematical problems spark such enthusiasm in these brilliant and famous people! Indeed, they were at the peak of their careers then. They "conjectured and proved", they founded schools, initiated new theories, published influential books and articles, and traveled all over the world. They lectured at every important university from Beijing to Calgary. They had a worldly air about them. They owned fine tweed jackets, listened to pocket radios, and wore shoes that required no laces! Such things do not escape a teenager's attention.

5 Against the Supreme Fascist

Erdős's way of thinking was very close to the ideas of the French Enlightenment. He believed deeply that all great problems facing mankind have rational solutions. In his own way, he always tried to do everything to further the truth. As a profane extension of an idea from Anatole France's novel, *Revolt of the Angels*, he liked to call Fate the "Supreme Fascist". (Fascism had been the determining experience of his younger years, so it is not surprising that he used it as a metaphor for Evil.) He said, more or less as a joke that life was a game, in which our opponent was the Supreme Fascist (S.F.). The S.F. gets two points if we do something mean and one point if we fail to do good. In this game, only the S.F. can win, but it should be our goal to keep his score low. Paul Erdős was certainly an international grandmaster at this game. When anyone needed his help, he was there for them. He disbursed money among needy friends and distant relatives. He told them to keep it as long as they wished, and one day, when they were rich, they could return it. In the same spirit, he assumed legal and financial responsibility for a girl he had never met before, who wanted to move to Vienna to fulfil her dream of becoming a musician. (She succeeded!) After the sudden death of his mother in 1971, Erdős's apartment in Budapest was used mostly by visiting mathematicians, old friends like Lee Lorch, Gábor Szegő, György Szekeres, or colleagues with temporary housing problems.

He often mentioned that half a century ago Americans tended to be far less tolerant and open-minded about cultural differences and alternative lifestyles than they are nowadays. Princeton had several elegant "restricted" neighborhoods, where no Jews or Italians were allowed to reside. Many colleagues at the University and at the Institute felt uneasy about the presence of this outspoken Hungarian Jew who approached them with a kind of childish openness, and often showed up uninvited in their offices to discuss a mathematical question. They could not understand why he never opened a checking account or owned a house or applied for a proper teaching job. He kept producing the most beautiful results, among them his ground-breaking papers on probabilistic number theory with A. Wintner and M. Kac, and on approximation theory with Turán, yet the Institute for Advanced Study did not renew his fellowship for the year 1939. John von Neumann, like other leading mathematicians, was aware, certainly, of Erdős's extraordinary qualities and achievements, but did not lift a finger to help him. It is very likely that these events also contributed to Erdős's willingness to support those slightly quirky or eccentric mathematicians who were known to be gifted, but whom most people tried to avoid for this very reason. His political experience had taught him to raise his voice every time he believed an organization, government, or authority was infringing on an individual's private life and freedom. He highly respected his Communist friends who took part in the underground resistance against the Nazis, and lived through the infernos of Fascism and Stalinism. He also respected American civil rights activists, many of whom paid a very high price for their dreams and ideals. He often took a stand on much smaller issues, as well. Recently, he resigned from his honorary doctorate from the University of Waterloo, in protest against the unfair dismissal of a colleague.

The sensational mathematical event of 1948 was Atle Selberg's and Paul Erdős's discovery in Princeton of an elementary proof of the celebrated Prime Number Theorem. This result, conjectured by Gauss, and first established simultaneously by Hadamard and de la Vallée Poussin using heavy analytical tools, states that the number of primes smaller than n is asymptotically equal to $n/\log n$. Selberg and Erdős agreed to publish their results back to back in the same journal, indicating precisely their individual contributions. Unfortunately, it happened otherwise. Allegedly, not long after they made their discovery, it came to Selberg's ears that 'Erdős and someone else' had found an elementary proof of the Prime Number Theorem. He was so upset that he rushed ahead with the publication of his paper. Two years later, partly for this proof, he received the Fields Medal (the highest prize in mathematics, comparable only to a Nobel Prize), which he more than deserved. Erdős also published his contribution, for which the American Mathematical Society awarded him the (somewhat less prestigious) Cole Prize in 1951. Erdős's role in the elementary proof of the Prime Number Theorem has been underestimated by the mathematical establishment. His colleagues at Princeton treated him especially coldly.

Anyone else would have been depressed by such developments, and discouraged from doing joint work. But Paul did not care too much whether he deserved a Fields Medal. He was distressed only because he lost forever one of his greatest collaborators, A. Selberg. He continued to travel perpetually around the world, producing an infinite stream of questions, conjectures, and proofs. I have never met a more generous and noble-minded person than Paul. Wherever he went, his route was marked by a trail of mathematical ideas and discoveries. Like a missionary, he found followers everywhere. His famous problem surveys provide enough material, ideas, and open questions for another generation of researchers. I have no doubt that his mathematical diaries, kept for almost six decades, also hold a few scientific surprises.

According to Erdős, a wicked pastime of the "Supreme Fascist" is to try to keep secret the most beautiful mathematical theorems and laws of nature. But in this awesome struggle, Erdős won some impressive victories against him. He found dozens of theorems and proofs which will be surely cited for centuries, if mankind survives. One of them is the following inductive argument, which proves that the product of all primes that do not exceed n is at most 4^n.

If $n \leq 4$, then the statement is obviously true. Assume that we have already verified the statement for all positive integers smaller than n, and we want to prove it for n. We can assume that n is odd. The product of all primes that do not exceed n can be written as

$$\prod_{p\leq n} p = \prod_{p\leq \frac{n+1}{2}} p \cdot \prod_{\frac{n+1}{2}<p\leq n} p.$$

By induction hypothesis, the first factor on the right-hand side is at most $4^{(n+1)/2}$. And now comes a brilliant idea, a surprising step so characteristic of Erdős. Notice that the second factor can be bounded from above by $\binom{n}{\frac{n+1}{2}}$, because every prime larger than $(n+1)/2$ occurs in the factorization of this binomial coefficient. Therefore,

$$\prod_{p\leq n} p \leq 4^{\frac{n+1}{2}} \cdot \binom{n}{\frac{n+1}{2}} \leq 4^{\frac{n+1}{2}} \cdot 2^{n-1} = 4^n,$$

as required. It is worth noting that the above statement immediately implies a weaker version of one side of the Prime Number Theorem: the number of primes smaller than n is at most $cn/\log n$, where c is a suitable constant.

Paul was invited to lecture at so many places that he could not accept all the invitations he received. His mother would remind him: "Palkó, not even you can be in two places at once!".

But, in the end, this is exactly what happened.