

Course Syllabus

MA-UY 4424: Numerical analysis

Lecture: Tues & Thurs 2:00pm - 3:15pm, Jacobs 473

Recitation: Fri 1:3pm - 3:45pm, Rogers 503

Course website: <https://cims.nyu.edu/~oneil/na20>

Prerequisites: Grade C or better in Calculus III and Linear Algebra. MATLAB will be used as the primary language and you will be expected to master it (or Python, or Julia) in the first few weeks of the course. Before you come to class, obtain MATLAB via the NYU-wide license:

<https://www.nyu.edu/life/information-technology/getting-started/software/matlab.html>

Description: This course will serve as an introduction to modern numerical analysis and will cover subjects such as the solution of systems of nonlinear equations, numerical linear algebra, numerical differentiation and integration, interpolation, Monte Carlo methods, and numerical methods for ordinary differential equations. The stability and accuracy of all the previous methods will also be analyzed in the context of floating-point arithmetic. The course will have a focus on the analysis of numerical methods, but also require you to use numerical software (MATLAB, Python, or Julia). If you are not familiar with any of these languages, the recitation will give an introduction to MATLAB during the first weeks.

Objectives: By the end of the course, students will be able to implement standard numerical algorithms for finding roots of functions, solving systems of equations, solving ordinary differential equations, and computing standard matrix factorizations. Choosing the most appropriate numerical algorithm for a given problem and successfully implementing a stable algorithm are goals of this course. The methods covered should be of use in mathematics, physics, computer science and all fields of engineering.

Materials: The course will loosely follow *An Introduction to Numerical Analysis* by Süli and Meyers. Other recommended texts are the following:

- Burden and Faires, *Numerical Analysis*
- Dahlquist and Björck, *Numerical Methods*
- Greenbaum and Chartier, *Numerical Methods: Design, Analysis, and Computer Implementation of Algorithms*
- Trefethen and Bau, *Numerical Linear Algebra*

Grading: The overall course grade will be determined based on homework, two preliminary exams, and a final exam. The following breakdown will be used to compute an overall numerical grade, which will then be translated to a letter grade:

- 10% Homework
- 25% Preliminary exam 1
- 25% Preliminary exam 2
- 40% Final exam

Homework will be assigned roughly every other week and due by the start of class on the day on which it is due. A PDF of the homework may be sent in lieu of a hard-copy but the instructor must receive it (electronically) by the start of class. No late homework is accepted without **prior** approval from the instructor, and in those cases, generally exemptions will only be made for medical reasons with documentation. Students are encouraged to work together on their homework, but each student must write-up and submit the assignments independently.

Please see the below link for a list of policies that will be enforced at Tandon:

<https://math.nyu.edu/dynamic/sites/tandon/bs-students/exam-and-homework-policies/>

NYU's academic integrity policies will be strictly enforced for homework assignments. While the lectures and textbook are meant to be focused on theoretical numerical analysis, computing on your own will form an essential part of the learning process and your own applied mathematics training. You are welcome to discuss problems, talk to your colleagues and consult other sources when you get stuck, **as long as you explicitly acknowledge any help that you receive from any source**, but **you must write all of your homework solutions and every line of code yourself** (including making your own figures). There is no substitute for debugging your own code; looking at or copying someone else's code is not the same. All code must be commented in detail by you. If your code is not commented, you will receive no credit for that problem. If your code or comments are copied from someone else, each of you will receive a zero for the assignment.

Weekly schedule:

1. Solving nonlinear equations: bisection, secant method, Newton's method
2. Fixed point iterations: Contraction Mapping Theorem
3. IEEE floating point arithmetic, round-off error vs. approximation error
4. Numerical linear algebra: Gaussian elimination, LU factorization, norms
5. Numerical linear algebra: Gram-Schmidt, QR factorization, least squares
6. Numerical linear algebra: The SVD, condition numbers, Jacobi rotations
7. Numerical linear algebra: Eigenvalue/vectors, power and inverse-power iterations
8. Interpolation: Lagrange interpolation, barycentric form
9. Interpolation: Chebyshev interpolation
10. Numerical integration: Trapezoidal rule, Newton-Cotes formulae
11. Numerical integration: Gaussian quadrature and orthogonal polynomials
12. Numerical methods for ODEs: Initial Value Problems
13. Monte Carlo: Generating random variables, convergence rates, applications
14. Fast Fourier Transform: Fourier series, quadrature, computational complexity