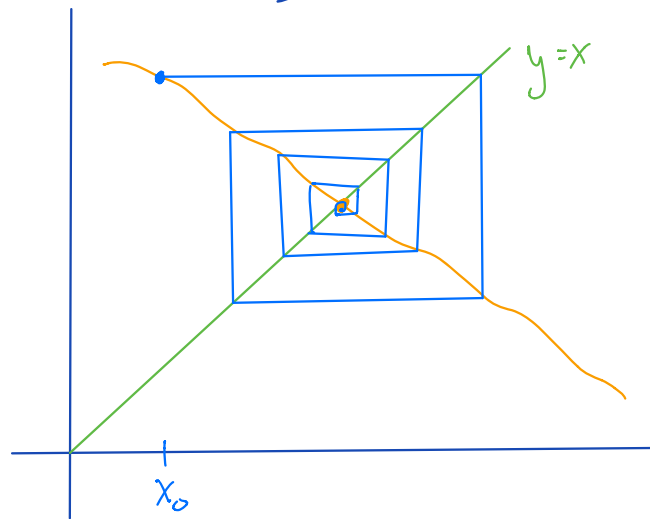


Another Example:

The above examples all had $g' > 0$.

What happens when $g' < 0$, but $|g'| < 1$?

We get oscillatory behavior:



We've seen the relationship between $g'(s)$ and the convergence/stability of $x_{k+1} = g(x_k)$. This can be made into the following theorem.

Theorem Suppose g is continuously differentiable in a neighborhood of its fixed point ξ , and let $|g'(\xi)| > 1$. Then, $x_{k+1} = g(x_k)$ does not converge to ξ for any $x_0 \neq \xi$.

Sketch of Proof: Let $I_\delta = [\xi - \delta, \xi + \delta]$ be the δ -neighborhood of ξ . Then by the MVT,

$$\begin{aligned} |x_{k+1} - \xi| &= |g(x_k) - g(\xi)| \\ &= |g'(\eta_k)| |x_k - \xi| \quad \text{for some } \eta_k \\ &\geq L |x_k - \xi| \quad \text{with } L > 1. \end{aligned}$$

So the sequence is diverging from ξ , and eventually will exit the interval I_δ .

Example: Some functions have both
stable and unstable fixed points:

