Homework 7

Due: 2:00pm Mar. 31st, 2016

Each problem is worth 10 points.

Exercise 1 [Richardson extrapolation]: Using the finite difference approximation

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

perform Richardson extrapolation using $h = 0.2$, 0.1, and 0.05 for the function $f(x) = \sin x$ at $x = \pi/6$. That is to say, use the results of Richardson extrapolation for $h = 0.2 \to 0.1$ with the result of Richardson extrapolation for $h = 0.1 \to 0.05$ to perform one additional Richardson extrapolation. Make a table of the results. What order of approximation do you obtain after these two steps of Richardson?

Exercise 2 [Newton-Cotes] Derive the Newton-Cotes quadrature rule for

$$\int_0^1 f(x)\, dx$$

using the nodes $x = 0$, $1/3$, $2/3$, and 1.

Exercise 3 : Find the formula of the form

$$\int_0^1 f(x)\, dx \approx w_0 f(0) + w_1 f(1)$$

that is exact for all functions of the form

$$f(x) = ae^x + b\cos\left(\frac{\pi x}{2}\right),$$

with a, b constants.

Exercise 4 [Mid-point rule]: The mid-point quadrature rule is given by:

$$\int_a^b f(x)\, dx \approx (b-a) f\left(\frac{a+b}{2}\right).$$

(a) Show that this quadrature rule is exact if f is a constant or linear function.

(b) What is the approximation error in the mid-point rule above?