Theory of Probability

Nov 2, 2020

Normal approximation to the binomial pmf:

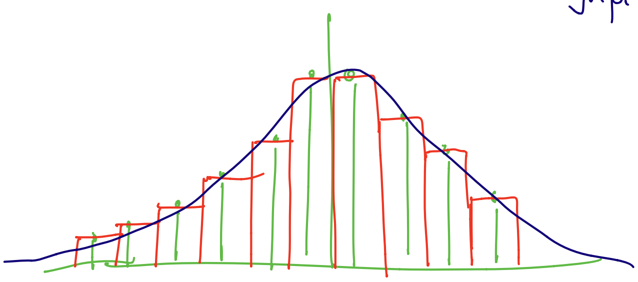
Let $X \sim \text{binomial}(n, p)$.

$$P[X = k] = \binom{n}{k} p^k (1-p)^{n-k}$$

continuity correction.

$$= P\left[k - \tfrac{1}{2} \le X \le k + \tfrac{1}{2}\right]$$

$$= P\Bigg[\frac{k-\frac{1}{2}-np}{\sqrt{np(1-p)}} \le \underbrace{\frac{X-np}{\sqrt{np(1-p)}}}_{} \le \frac{k+\frac{1}{2}-np}{\sqrt{np(1-p)}}\Bigg]$$



by the DeMoivre-Laplace approximation, this $\rightsquigarrow N(0,1)$ as $n \to \infty$.

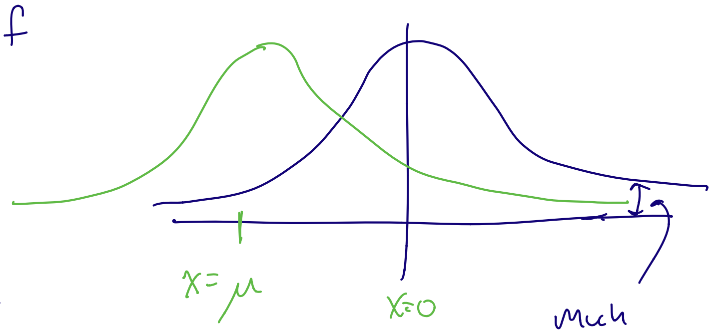
$$\approx P\left[\frac{k-\frac{1}{2}-np}{\sqrt{np(1-p)}} \le Z \le \frac{k+\frac{1}{2}-np}{\sqrt{np(1-p)}}\right]$$

$$= \Phi\left(\frac{k+\frac{1}{2}-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{k-\frac{1}{2}-np}{\sqrt{np(1-p)}}\right)$$

Cauchy Distribution has pdf

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

$$f(x; \mu) = \frac{1}{\pi} \frac{1}{1+(x-\mu)^2}$$

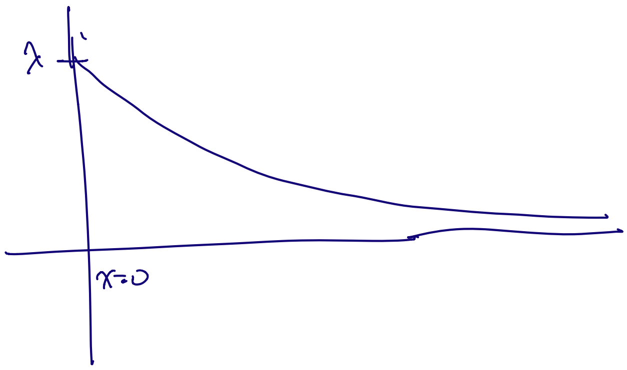


much thicker than normal.

$$F(x) = \int_{-\infty}^{x} f(x;\mu)\, dx$$

Exponential R.V.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



Modeling:

- radioactive decay
- rare event modeling
- waiting times.

Gamma R.V.

$$f(x) = \begin{cases} \dfrac{\lambda e^{-\lambda x} (\lambda x)^{\alpha-1}}{\Gamma(\alpha)} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Modeling

- positive quantities
- stock prices