Prop. 4.4 : Inclusion - Exclusion

\[ P(A \cup C) = P(A) + P(C) - P(A \cap C) \]
\[ P(S) = 1 \]
\[ P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) = 0 \]

Problem 9 from Ch. 2:

- 24% carry AMEX
- 61% carry Visa
- 11% carry both cards.

What percent carry AMEX or Visa? \[ 24\% + 61\% - 11\% = 74\% \]

What percent carries AMEX but not Visa? 13%
Theoretical Exercise 11

Let \( P(E) = 0.9 \)
\[ P(F) = 0.8 \]

Show that \( P(E \cap F) = P(EF) \geq 0.7 \).

By Inclusion-Exclusion,
\[
P(E \cup F) = P(E) + P(F) - P(EF)
\]
\[
= 0.9 + 0.8 - P(EF)
\]
\[ P(E \cup F) = 1.7 - P(EF) \]

Since \( P(E \cup F) \leq 1 \), it must be that \( P(EF) \geq 0.7 \).

More generally, we have Bonferroni’s Inequality:
\[ P(EF) \geq P(E) + P(F) - 1 \]

\[ P(E \cup F) = P(E) + P(F) - P(EF) \]

Since \( P(E \cup F) \leq 1 \) we have that

\[ P(E) + P(F) - P(EF) \leq 1 \]

\[ \Rightarrow \]
\[ P(EF) \geq P(E) + P(F) - 1 \].
Problem 7 from Ch. 2

15 members of a soccer team, each is either blue collar/white collar, and either R/D/I.

How many outcomes are
(a) in the sample space?

15

Person 1  Person 2  Person 3  ...  15

- BR
- BD
- BI
- WR
- WD
- WI

(b) How many outcomes are in the event that at least one of the team members is a blue collar worker?

Complement: None of the team members are blue collar workers.

\[ 6^{15} - \text{ # outcomes when none are blue collar.} \]
(c) How many outcomes are in the event that none of the team members consider themselves independent? $4^{15}$