Theory of Probability

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Random Variable: Is a function defined on a sample space.

Notation: Capital letters are random variables: X, Y, Z

Possible values of the R.V. are lower case: x, y, z

Discrete Random Variables

A random variable that can take on at most a "countable" number of values is a discrete random variable.

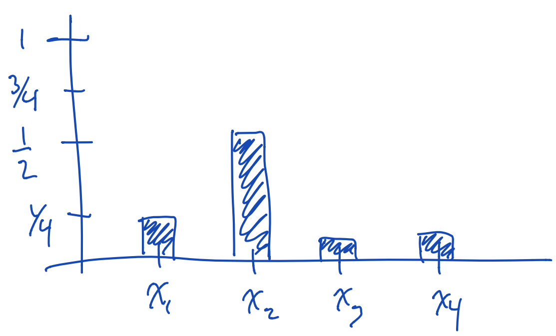
Probability Mass function (density): $p(a) = P(X = a)$.

Cumulative Distribution function: $F(x) = P(X \le x)$.

Note: If X can take on values $x_1, x_2, x_3, \ldots$

then $\sum_{i=1}^{\infty} p(x_i) = 1$.

Graphical Depiction of probability mass function: bar chart.



$p(x_1) = \frac{1}{4}$

$p(x_2) = 1/2$

$p(x_3) = 1/8$

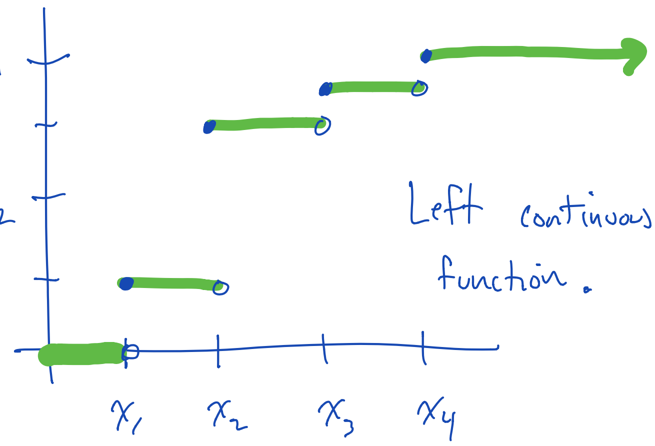
$p(x_4) = 1/8$

In terms of the mass function p, the cumulative distribution function F is:

$$F(x) = P(X \le x) = \sum_{x_i \le x} p(x_i)$$

Plot of F:

$$F(x) = \begin{cases} 0 & \text{if } x < x_1 \\ 1/4 & x \in [x_1, x_2) \\ 3/4 & x \in [x_2, x_3) \\ 7/8 & x \in [x_3, x_4) \\ 1 & x \in [x_4, \infty) \end{cases}$$



Left continuous function.

Expected Values

The expected value is the weighted average of a random variable.

$$E(X) = \sum_{i=1}^{\infty} x_i \cdot P(X = x_i)$$

$$= \sum_{i=1}^{\infty} x_i \, p(x_i).$$

Example: Rolling a die.

$x_1 = 1$ $\quad P(X = x_i) = 1/6 = p(x_i)$

$x_2 = 2$

$\vdots$

$x_6 = 6$

$$E(X) = \sum_{1}^{6} x_i \, p(x_i)$$

$$= 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = \frac{21}{6} = \frac{7}{2}$$