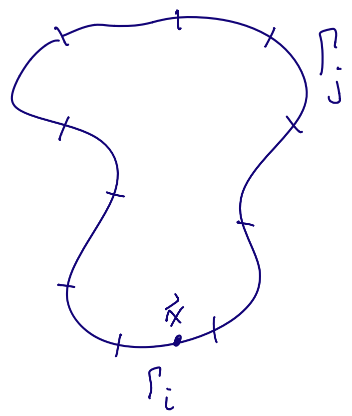
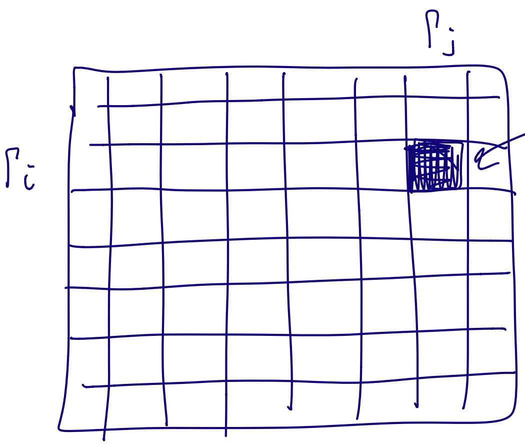
Fast Solvers   Nov 11, 2020

Proxy surfaces & skeletonization



Consider the operator which maps functions on Γ_j to functions on Γ_i :

$$\varphi(\vec{x}) = \int_{\Gamma_j} G(\vec{x}, \vec{x}')\, \sigma(\vec{x}')\, ds(\vec{x}')$$



matrix elements are

$$w_j\, G(\vec{x}_i, \vec{x}_j).$$

The continuous operator : $S_{ij} : L^2(\Gamma_j) \to L^2(\Gamma_i)$

is compact.

$$\varphi(\vec{x}) = S_{ij}\, \sigma(\vec{x}) = \int_{\Gamma_j} G(\vec{x}, \vec{x}')\, \sigma(\vec{x}')\, ds(\vec{x}')$$

In this case, G is smooth.

Let $\vec{x}_{jk}, w_{jk}$ be a quadrature for smooth functions on Γ_j

$\vec{x}_{ik}, w_{ik}$ be .... on Γ_i.

Let $\varphi_{ik} = \sqrt{w_{ik}}\, \varphi(\vec{x}_{ik})$ "L^2 embedding".

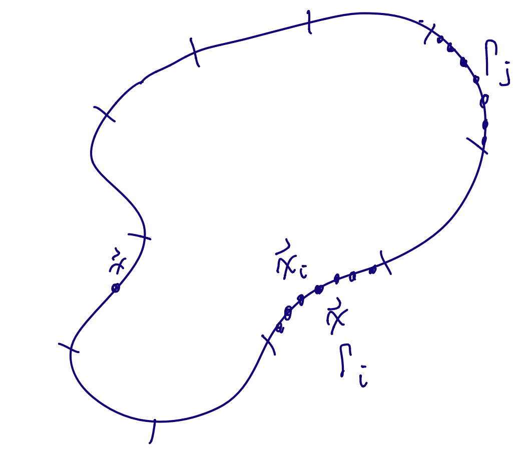
$\sigma_{jk} = \sqrt{w_{jk}}\, \sigma(\vec{x}_{jk})$

so that
$$\|\vec{\varphi}_i\|^2_{\ell^2} = \sum_k w_{ik} |\varphi(\vec{x}_{ik})|^2$$
$$\approx \|\varphi_i\|^2_{L^2}$$

Let $A_{kk'} = \sqrt{w_{ik}}\, G(\vec{x}_{ik}, \vec{x}_{jk})\sqrt{w_{jk'}}$. then A has eigenvalues which converge to the eigenvalues of S_{ij}.

$$\Rightarrow \|S_{ij}\|^2_{L^2} \approx \|A\|^2_{\ell^2}.$$

Proxy Surface



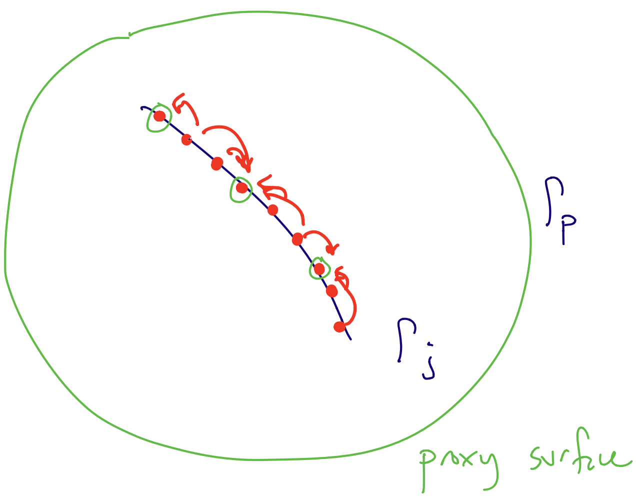
$$\varphi(\vec{x}) = \int_\Gamma G(\vec{x}, \vec{x}')\,\sigma(\vec{x}')\,ds(\vec{x}')$$
$$\approx \sum_j G(\vec{x}, \vec{x}_j')\,\sigma_j\, w_j$$

Any $\varphi(\vec{x})$ is just a linear combination of $G(\vec{x}, \vec{x}_j')$.

$$\begin{pmatrix} \varphi(\vec{x}_1) \\ \vdots \\ \varphi(x_m) \end{pmatrix} = \underbrace{\begin{pmatrix} w_j\, G(\vec{x}_i, \vec{x}_j') \end{pmatrix}}_{\text{Compute ID of this matrix. } A.} \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{pmatrix}$$

$$A = A_{skel}\, R$$

$$= m\left\{ \underbrace{\begin{pmatrix} w_{j_1} G(x_1, x'_{j_1}) & w_{j_2} G(x_1, x'_{j_2}) & \cdots & w_{j_k} G(x_1, x'_{j_k}) \\ \vdots & \vdots & & \\ & & & \end{pmatrix}}_{k \text{ columns}} \right. \underbrace{R}_{k \times n \text{ matrix.}}$$



R spreads densities on non-skeleton points to the skeleton points.

The sums inside generate φ on Γ_p.

Green's 3rd Id:

$$\varphi(\vec{x}) = D_{\Gamma_p}\varphi - S_{\Gamma_p}\frac{\partial \varphi}{\partial n}$$