Integral Equations and Fast Algorithms Course Project

Due: 11:59pm December 22nd, 2017

You are free to implement any parts of this project in the language of your choice: `C`, `Fortran`, `Matlab`, `Python`, etc. The only requirement is that you are able to produce plots when necessary, and do *not* use high-level routines (e.g. pre-built packages/solvers, etc.) when the assignment requires that you implement those yourself.

Your code should be *easily* compiled (if necessary) and executed (on Linux or OS X) with commands such as `gfortran myproject.f; ./a.out`. The simpler the better.

Your final grade in the course will be determined by this project as well as classroom interactions.

1. Choose a PDE and boundary value problem (BVP). For example, *the interior Laplace Dirichlet problem*:
$$\begin{aligned} \Delta u &= 0 \qquad \text{in } \Omega \subset \mathbb{R}^2, \\ u &= f \qquad \text{on } \partial\Omega. \end{aligned} \tag{1}$$
(Don't choose this one, as we did all the required proofs in class.)

   Don't forget to state all the required conditions that make the problem well-posed (e.g. any radiation/decay conditions on the solution for exterior problems).
2. Prove existence/uniqueness of the solution to the BVP.
3. Derive an integral equation formulation of (1).
4. Using the Fredholm alternative, prove invertibility of the integral equation you derived.
5. Numerically implement (this means write a program) to solve the integral equation in part 3 (and therefore the BVP). This will require you to:
   - Pick a discretization method and quadrature rule.
   - Solve the resulting linear system.
   - Show how to test the solver and verify the order of convergence.
6. Show a plot of the solution for a test problem.