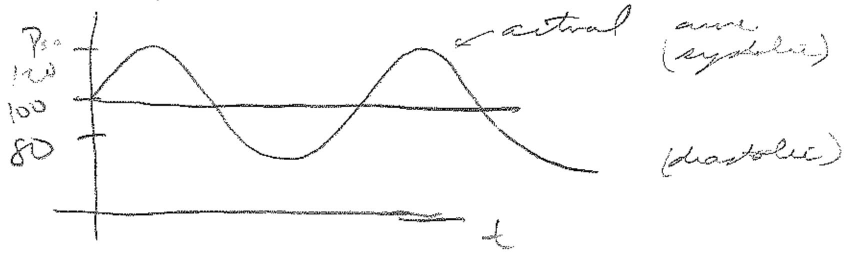
09/29/2022

Dynamics of arterial pulse

Our model has been kind of confusing - when we think about the circulation, we usually think about beats and pulses, but we've been doing steady state modeling. In particular, we have been modeling P_{sa} as t → ∞.



Def The pulse pressure is the difference between the systolic and diastolic pressures

Normal conditions
$P_{sa}^{(sys)}$ = pressure in left ventricle in systole (since aortic valve is open)
$P^{(dia)}$ in the ventricle < $P_{sa}^{(dia)}$ because aortic valve is closed.

Consider the heart as a compliance vessel not in steady state

$$\frac{dV}{dt} = Q_1 - Q_2$$

Compliance: $\frac{d(CP+V_d)}{dt} = Q_1 - Q_2$

Assuming C is constant, get

$$C\frac{dP}{dt} = Q_1 - Q_2$$

Apply this to systemic arteries $P = P_{sa}$, $C = C_{sa}$, $Q_1 = Q_L$ (output of left heart), $Q_2 = Q_s = \frac{P_{sa} - P_{sv}}{R_s} \approx \frac{P_{sa}}{R_s}$.

Thus we have the overall ODE:

$$\boxed{C_{sa}\frac{dP_{sa}}{dt} = Q_L - \frac{P_{sa}}{R_s}}$$

Exercise: Solve this eqn during diastole ($Q_L = 0$). That is, solve $C_{sa} \frac{dP_{sa}}{dt} = -\frac{P_{sa}}{R_s}$ with $P_{sa}(0) = P_{sa}^{(0)}$.

Solution: $P_{sa}(t) = P_{sa}(0)\exp\left(-t/(R_s C_{sa})\right)$
with $P_{sa}(0)$ to be determined
to find $P_{sa}(0)$, consider systole. Assume stroke volume.
Vstroke $= \Delta V_0$ ejected from heart at once. Cannot use ODE since we'd have a δ-function. Instead we know from

$$V = CP + V_d,$$

$$\Delta V_0 = C_{sa}\,\Delta P_{sa}$$ (change in arteries by sudden increase in volume).

Now suppose this is periodic with period $T = 1/F$.
then

$$\Delta P_{sa} = P_{sa}(0) - P_{sa}(T) = P_{sa}(0)\left[1 - \exp(-T/R_s C_{sa})\right]$$

So $$P_{sa}(0) = \frac{\Delta V_0}{C_{sa}}\left(1 - \exp(-T/R_s C_{sa})\right)^{-1}$$

and $$P_{sa}(T) = \frac{\Delta V_0}{C_{sa}}\left(1 - \exp(-T/R_s C_{sa})\right)^{-1} \cdot \exp\left(-\tfrac{T}{R_s C_{sa}}\right)$$

to simplify notation let $\theta = \exp(-T/R_s C_{sa})$.

$$P_{sa}(0) = \frac{\Delta V_0}{C_{sa}(1-\theta)} \qquad P_{sa}(T) = \frac{\Delta V_0\,\theta}{C_{sa}(1-\theta)}$$

the pulse pressure is the difference of these two.
Now, what is the mean P_{sa}, and how does it compare to a steady state model?
Mean of a periodic function f: $\langle f \rangle = \frac{1}{T}\int_0^T f(t)\,dt$.
Exercise: using the definition, solve for $\langle P_{sa} \rangle$

$$\langle P_{sa} \rangle = \frac{\Delta V_0}{T C_{sa}(1-\theta)} \int_0^T \exp(-t/R_s C_{sa})$$

$$= -\frac{\Delta V_0 R_s}{(1-\theta)T} \exp(-t/R_s C_{sa})\Big|_0^T$$

$$= -\frac{\Delta V_0 R_s}{T(1-\theta)}\left[\theta - 1\right] = \frac{R_s \Delta V_0}{T}$$

So $\langle P_{Sa} \rangle = \dfrac{R_S \, \omega V_0}{T}$

But $\omega V_0 / T$ is the cardiac output Q,

so $P_{Sa} = R_S Q$, which is the steady state eqn

with P_{SV} neglected. Thus, the quantities in the

steady state are time averages of the dynamic quantities!