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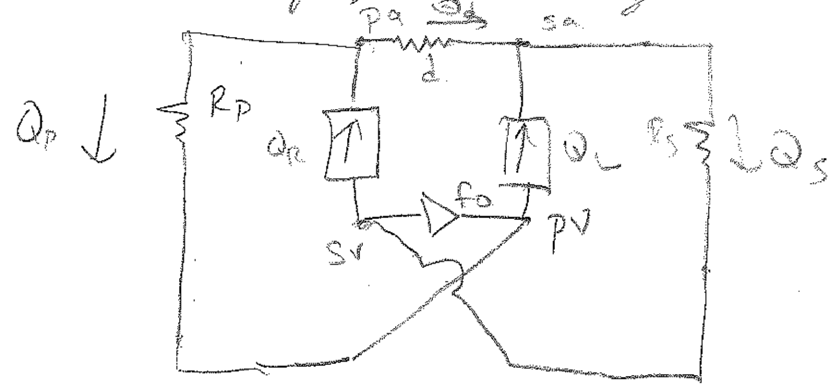
Changes in the circulation at birth.

Before birth, the lungs of a baby's circulation are collapsed, meaning they present high resistance to blood flow

At the first breath, the lungs regain their normal resistance

How does the body compensate for higher resistance? add ducts to shunt blood away from the lungs.



First see the old circulation drawn in a new way. Then two additional shunts (highlighted).

fo = foramen ovale: opening in the wall of the heart ensuring that flow goes from ⟶

d = ductus arteriosus, connects pulmonary and systemic arteries near the heart.

These connections shunt blood away from the lungs.

<u>Model</u> - based on uncontrolled circulation

$Q_R = K_R P_{sv}$

$Q_L = K_L P_{pv}$

$Q_p = (P_{pa} - P_{pv}) / R_p$

$Q_S = (P_{sa} - P_{sv}) / R_S$

$Q_d = (P_{sa} - P_{pa}) / R_d$ (model of ductus as resistance vessel)

Foramen - ideal valve

OPEN: $Q_f \geq 0$ $P_{sv} = P_{pv}$

CLOSED: $Q_f = 0$ $P_{pv} \geq P_{sv}$ (blocks blood flow in wrong direction)

Notice: we are ignoring volume here, so the unknowns are

$Q_R, Q_L, Q_p, Q_S, Q_d, Q_f, P_{pa}, P_{pv}, P_{sv}, P_{sa}$ (10)

We have 6 eqns. Need 4 more eqns.

<u>Exercise</u>: use conservation of volume at junctions to write 4 more eqns.

Blood flow at junctions

$$Q_R = Q_d + Q_p \qquad (1).$$
$$Q_L + Q_d = Q_S \qquad (2)$$
$$Q_f + Q_p = Q_L \qquad (3)$$
$$Q_S = Q_R + Q_f \qquad (4)$$

Only 3 of the eqns are independent, since

$$Q_R = Q_d + Q_p \quad (1).$$
$$(4)\ Q_R = Q_S - Q_f \overset{(2)}{=} Q_L + Q_d - Q_f \overset{(3)}{=} Q_f + Q_p + Q_d - Q_f$$
$$= Q_p + Q_d.$$

So we have 3 independent equations among 6 flows. Can choose 3 flows to specify the others in terms of.
Choosing Q_L, Q_d and Q_R:

$$\text{others } \begin{cases} Q_p = Q_R - Q_d & (5) \\ Q_S = Q_L + Q_d & (6) \\ Q_f = Q_L - Q_p = Q_L + Q_d - Q_R & (7) \end{cases}$$

$Q_f = Q_L + Q_d - Q_R$: shows importance of having 2 shunts. If $Q_L = Q_R$, then $Q_f = Q_d$. But if $Q_f = 0$, then an imbalance will be required for $Q_d > 0$ (to shunt blood away from lungs). This is important in development as heart should develop the same on both sides.

<u>Assumptions to solve model</u>

1) $K_R = K_L = K$ (2 sides of heart are same before birth)
2) Ductus wide open before birth
$$R_d = 0 \longrightarrow P_{sa} = P_{pa} := P_a$$
3) Assume foramen is also open
$$P_{sv} = P_{pv} = P_v.$$

[We will solve for Q_f and check this is indeed the case]

Model simplifies to

$$Q := K P_v \quad \text{on both sides} \quad [Q = Q_R = Q_L]$$
$$R_S Q_S = R_p Q_p = P_a - P_v.$$

Therefore
$$\boxed{\frac{Q_p}{Q_S} = \frac{R_S}{R_p}}$$

$Q_p = Q - Q_d, \quad Q_S = Q + Q_d$

$$\frac{Q - Q_d}{Q + Q_d} = \frac{R_S}{R_p}$$

<u>Exercise</u>: solve for Q_d/Q. then use (7) to solve for Q_f/Q.

$$QR_p - Q_d R_p =$$

$$R_p - \left(\frac{Q_d}{Q}\right) R_p = R_S + R_S\left(\frac{Q_d}{Q}\right)$$

$$\boxed{\frac{Q_d}{Q} = \frac{R_p - R_S}{R_p + R_S}}$$

Then using (7) with $Q_L = Q_R$, get $Q_f = Q_d$.

So $\boxed{\frac{Q_f}{Q} = \frac{R_p - R_S}{R_p + R_S}}$

Now we get Q_S and Q_p.

$$(6) \to \frac{Q_S}{Q} = 1 + \frac{Q_d}{Q} = 1 + \frac{R_p - R_S}{R_p + R_S} = \boxed{\frac{2R_p}{R_p + R_S} = \frac{Q_S}{Q}}$$

$$\frac{Q_p}{Q} = 1 - \frac{Q_d}{Q} = 1 - \left(\frac{R_p - R_S}{R_p + R_S}\right) = \boxed{\frac{2R_S}{R_p + R_S} = \frac{Q_p}{Q}}$$

If $R_p > R_S$, then $Q_f > 0$

take limit $R_p \to \infty$, $Q_S/Q = 2$

$Q_p/Q = 0$

At birth, lungs expand so that $R_p < R_S$. then we have $Q_f < 0$, so our solution is no longer self consistent. In HW you will do the reverse case.