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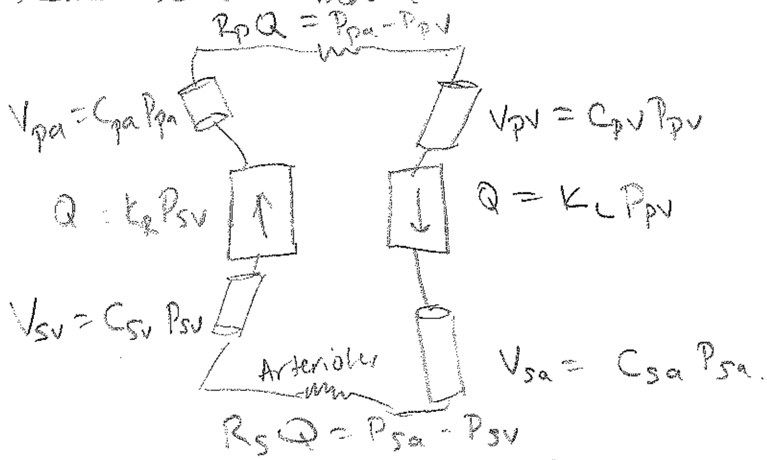
Analysis of circulation model

Remember our model:



$V_0 = V_{sv} + V_{pa} + V_{pv} + V_{sa}$

We solved it to find

$$Q = \frac{V_0}{T_{sa} + T_{sv} + T_{pa} + T_{pv}}$$

$$T_{sa} = \frac{C_{sa}}{K_R} + C_{sa} R_s$$

$$T_{sv} = C_{sv} / K_R$$

$$T_{pa} = C_{pa}/K_L + C_{pa} R_p$$

$$T_{pv} = C_{pv}/K_L$$

$[V_i = T_i Q]$

Today we will do same analysis

1) Balancing the 2 sides of the heart

What would happen in the model if K_R was suddenly reduced?

Q from the right heart ⇓

Transfer of volume into systemic circ

Raises P_{sv}, lowers P_{pv}

Result is right heart output increasing and left decreasing

In this model, blood is partitioned between the pulmonary and systemic circulations as

$$\frac{V_p}{V_s} = \frac{V_{pa} + V_{pv}}{V_{sa} + V_{sv}} = \frac{T_{pa} + T_{pv}}{T_{sa} + T_{sv}} = \frac{\frac{C_{pa} + C_{pv}}{K_L} + C_{pa} R_p}{\frac{C_{sa} + C_{sv}}{K_R} + C_{sa} R_s}$$

this is how the circulation volumes adjust to keep the cardiac output the same.

Suppose $K_L \to 0$, then $V_p / V_s \to \infty$

$K_R \to 0$, then $V_p / V_s \to 0$.

2) The need for external control mechanisms

Consider exercise: in exercise, arterioles dilate (where the O_2 is exchanged to CO_2). Then <u>R_s is reduced</u>

Cardiac output Q rises, while maintains the blood pressure.

Increase in $Q \propto F$ (roughly prop. to heart rate)

$\alpha_1^Y = 1$

What happens in the uncontrolled circulation model?

Exercise: Using parameter values

$R_s = 17.5$ $R_p = 1.79$, $C_{sa} = 0.01$ $C_{pa} = 0.00667$

$C_{sv} = 1.75$ $C_{pv} = 0.08$

$K_R = 2.8$ $K_L = 1.12$

$V_0 = 5$

a) Compute P_{sa} and Q.

b) Compute the same if R_s is reduced by 50%.

$$Q = \frac{V_0}{T_{sa} + T_{sv} + T_{pa} + T_{pv}} \qquad P_{sa} = \frac{V_{sa}}{C_{sa}} = \frac{T_{sa} Q}{C_{sa}}$$

	T_{sa}	T_{sv}	T_{pa}	T_{pv}	Q	P_{sa}
Base	0.1786	0.625	0.0179	0.0714	5.6	100
$R_s/2$	0.0911	0.625	0.0179	0.0714	6.2	56

10% increase in cardiac output

40% drop in arterial pressure!

This is BAD - not enough to model exercise.

Introduce a more systematic way to understand these results: sensitivity

Def If Y depends on X and X changes, then the sensitivity of Y to X is defined as

$$\sigma_{YX} = \frac{\Delta \log Y}{\Delta \log X} = \frac{\log Y' - \log Y}{\log X' - \log X} = \frac{\log (Y'/Y)}{\log (X'/X)}$$

[Units and base of log don't matter]

$$\left[\log_a x = \frac{\log_b x}{\log_b a} \right]$$

When $Y' \approx Y$, $X' \approx X$, get

$$\sigma_{YX} = \frac{\Delta Y / Y}{\Delta X / X}$$ - sensitivity is roughly ratio of relative changes.

let $Y = X^n$. What is σ_{YX}?

$Y' = (X')^n$

$$\frac{\log (Y'/Y)}{\log (X'/X)} = \frac{\log(X'/X)\, n}{\log(X'/X)} = n.$$

In particular, for a direct relationship $Y \propto X$, $\sigma_{YX} = 1$.
When $Y \propto 1/X$, $\sigma_{YX} = -1$

$$\sigma_{QR_s} = \frac{\log(Q'/Q)}{\log(R_s'/R_s)} = \frac{\log(6.2/5.6)}{\log(1/2)} = -0.15$$

$$\sigma_{P_{sa}R_s} = \frac{\log(56/100)}{\log(1/2)} = 0.84$$

Notice that $\sigma_{P_{sa}R_s} = 1 + \sigma_{QR_s}$ (*)
$0.84 \qquad -0.15$

MEANING: if we make the model P_{sa} less sensitive to R_s, we also make Q more sensitive to R_s (we want that) So we only need to worry about one thing

SKIP
Reason for (*).

$$P_{sa} \approx Q R_s$$

$$\log P_{sa} = \log Q + \log R_s$$

$$\partial \log P_{sa} = \partial \log Q + \partial \log R_s$$

$$\frac{\partial \log P_{sa}}{\partial \log R_s} = \frac{\partial \log Q}{\partial \log R_s} + 1$$

$$\sigma_{P_{sa}R_s} = 1 + \sigma_{QR_s}.$$

Neural control

The most extreme case: hold P_{sa} constant, so $\sigma_{P_{sa}R_s} = 0$, $\sigma_{QR_s} = -1$.
In our bodies, the systemic arterial pressure is maintained by a part of the nervous system called the baroreceptor loop (details in book). Its effect is to adjust the heart rate F to maintain target pressure P^*.
New unknown: F. Remember F was actually in the model before: $K_R = F C_R$
$K_L = F C_L$
C_R and C_L are diastolic compliances of L and R heart. $Q = F C_R P_{sv} = F C_L P_{pv}$