Maximum volume - end of diastole

$$V_{ED} = V_d + C_{diastole} P_V.$$

Minimum volume - end of systole

$$V_{ES} = V_d + C_{sys} P_a$$

Stroke volume

$$V_{stroke} = V_{ED} - V_{ES}$$
$$= C_{dia} P_V - C_{sys} P_a.$$

Approximation: $C_{sys} \approx 0$; so

$$V_{stroke} = C_{dia} P_V.$$

which is the model of either side of the heart that we will use.

Let F = heart rate; get cardiac output

$$Q = F V_{stroke} = F C_{dia} P_V.$$

Suppose F is constant and let

$K = F C_{dia}$, so $Q = K P_V$, where

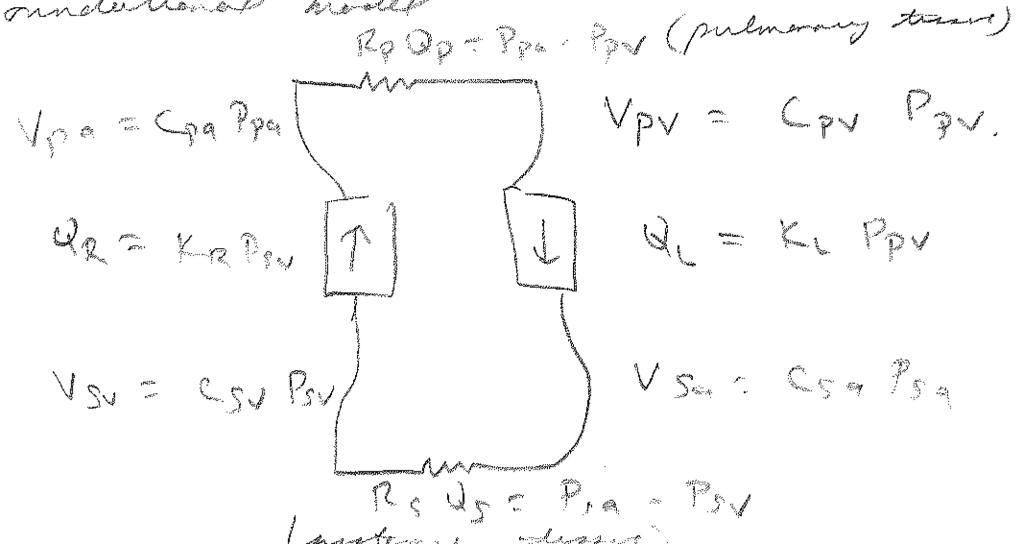
K is the "pump coefficient." The constant C_{dia} — greater in thin walled right V than in thick walled left ventricle, so we need two different expressions for left and right:

$$\boxed{\begin{aligned} Q_R &= K_R P_{SV} \\ Q_L &= K_L P_{PV} \end{aligned}}$$

<u>Mathematical model of the (uncontrolled) circulation</u>

Uncontrolled: no mechanisms to control it (more on this later)

Foundational model



Assumptions

1) Arteries & veins are compliance vessels with no dead volume
2) Systemic & pulmonary tissues are resistance vessels

Unknowns: Q_R, Q_L, Q_S, Q_P, P_{sa}, P_{sv}, P_{pa}, P_{pv}, V_{sa}, V_{sv}, V_{pa}, V_{pv}.

\# of eqns: 8 so far.

Need 4 additional eqns:

Total volume of blood given $V_{sa} + V_{sv} + V_{pa} + V_{pv} = V_0$.

Assume <u>steady state</u>, so $Q = Q_R = Q_L = Q_S = Q_P$

(volume/time entering = volume/time leaving)

Now have 9 eqns in 9 unknowns.

<u>Exercise</u>: solve the model

Strategy: 1) express all pressures in terms of Q
2) use compliance to get volumes in terms of Q.
3) Substitute total vol eqn & solve for Q.

1)
$$P_{sv} = Q / K_R$$
$$P_{pv} = Q / K_L$$
$$P_{sa} = P_{sv} + R_S Q = Q\left(\frac{1}{K_R} + R_S\right)$$
$$P_{pa} = P_{pv} + R_P Q = Q\left(\frac{1}{K_L} + R_P\right)$$

2)
$$V_{sv} = \frac{C_{sv} Q}{K_R} := T_{sv} Q \qquad V_{sa} = C_{sa} Q\left(\frac{1}{K_R} + R_S\right) := T_{sa} Q$$
$$V_{pv} = \frac{C_{pv} Q}{K_L} := T_{pv} Q \qquad V_{pa} = C_{pa} Q\left(\frac{1}{K_L} + R_P\right) := T_{pa} Q$$

3)
$$Q = \frac{V_0}{T_{sa} + T_{sv} + T_{pa} + T_{pv}}$$

$$V_i = \frac{T_i V_0}{T_{sa} + T_{sv} + T_{pa} + T_{pv}} \qquad P_i = \frac{T_i V_0}{C_i (T_{sa} + T_{sv} + T_{pa} + T_{pv})}$$