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transport of O_2

O_2 does not form a simple solution in blood. Recall that simple solutions obey

$$c = \sigma P$$

Where σ is the solubility of the gas.

For O_2, the relationship is given approximately by

$$c = \frac{c^* P^3}{P_*^3 + P^3}$$, where c^* and P_* are constants

Recall our simultaneous eqns for c_a and c_v.

$$V_A(c_I - c_A) = Q(c_a - c_v) \Rightarrow r(c_I - c_A) = c_a - c_v.$$

$$kTc_A = P_a = 1/\sigma\, c_a.$$ Not anymore!

Now we have

$$r(c_I - c_A) = c_a - c_v.$$

$$kTc_A = H(c_a).$$

Determine H

$$cP_*^3 + cP^3 = c^* P^3.$$

$$P^3(c^* - c) = cP_*^3.$$

$$\boxed{P = P_*\left(\frac{c_a}{c^* - c_a}\right)^{1/3}}$$

Then the arterial blood concentration is given by solving

$$r\left(c_I - \frac{P_*}{kT}\left(\frac{c_a}{c^* - c_a}\right)^{1/3}\right) = (c_a - c_v).$$

Nondimensionalize:

$$r\left(\frac{c_I}{c^*} - \frac{P_*}{kT}\left(\frac{c_a/c^*}{1 - c_a/c^*}\right)^{1/3}\right) = \left(\frac{c_a}{c^*} - \frac{c_v}{c^*}\right)$$

Put another way.

$$-r\left(\frac{c_I}{c^*}\right) - \left(\frac{c_v}{c^*}\right) + \frac{c_a}{c^*} + \frac{rP_*}{c^* kT}\left(\frac{c_a/c^*}{1 - c_a/c^*}\right)^{1/3} = 0$$

this is a nonlinear equation for c_a/c^*

Numbers: $c_I/c^* = 1$, $c_v/c^* = 0.4$, $P_*/kTc^* = 1/6$.

$$\Rightarrow -r - 0.4 + \frac{c_a}{c^*} + \frac{r}{6}\left(\frac{c_a/c^*}{1 - c_a/c^*}\right)^{1/3} = 0.$$

Aside: how to solve nonlinear eqns — do iteratively

Find x_{k+1} s.t. $f(x_{k+1}) \approx 0$.

Linearize $f(x_{k+1}) = f(x_k) + f'(x_k)(x_{k+1} - x_k) = 0$

Newton's method $\boxed{x_{k+1} = \frac{-f(x_k)}{f'(x_k)} + x_k}$

Here $f(x) = -r - 0.4 + x + \frac{r}{6}\left(\frac{x}{1-x}\right)^{1/3}$

$$f'(x) = 1 + \frac{1}{6}\cdot\frac{1}{3}\left(\frac{x}{1-x}\right)^{-2/3}\frac{(1-x)+x}{(1-x)^2}$$

$$= 1 + \frac{r}{18}\left(\frac{x}{1-x}\right)^{-2/3}\cdot\frac{1}{(1-x)^2}$$

$$= 1 + \frac{r}{18}\,\frac{1}{x^{2/3}(1-x)^{4/3}}$$