

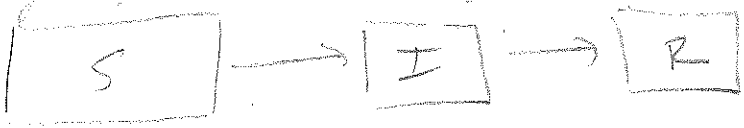
Introduce to compartment models for infectious disease

Infectious diseases - ~~pathogen~~ passed between individuals
 Pathogen can be divided into microparasites and macroparasites
 Each of these have their own direct transmission (Diseases central - covid!) and indirect transmission (pathogen survives outside of its host - like some diseases)

We will focus on macroparasites directly transmitted infectious diseases
Compartment models

Take a population and divide into compartments based on an individual's status simplest model

- 1) Pop. total is closed (no deaths or births)
- 2) Only infection recovery occurs



Let $\lambda S =$ rate of new infections. $\lambda =$ force of infection, per capita rate at which susceptible acquire infection
 Infection must be related to I, but how?

1) Density dependent $\lambda = \beta I$, where $\beta =$ transmission rate. β components: contact rate \times transmission probability

$I =$ # infected
 $\lambda I =$ new infections / time
 $\lambda =$ new infections / (susceptible \times time)

$\beta =$ new infection / (susceptible \times time (infected people) \times contact rate)

2) Frequency dependent transmission. $\lambda = \beta \frac{I}{N}$. λ is independent of pop size depends on fraction of infections

$\beta =$ new infections / (susceptible \times time \times person (fraction of people infected))
 $\beta =$ rate when everyone is infected
 $\lambda = \beta \frac{I}{N}$ (we will see this)

Recovery - let γ be recovery rate $1/\gamma =$ average time spent infected (e.g. 14 days rate is $1/14$ people/day)

With these assumptions, the ODE model is given by

$$\frac{dS}{dt} = -\beta \frac{I}{N} S$$

$$\frac{dI}{dt} = \beta \frac{I}{N} S - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

$$N = S + I + R = \text{constant}$$

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

Often useful to nondimensionalize state variables

$$\bar{S} = S/N \quad \bar{I} = I/N, \quad \bar{R} = R/N$$

gives simple model

$$\frac{d\bar{S}}{dt} = -\beta \bar{I} \bar{S}$$

$$\frac{d\bar{I}}{dt} = \beta \bar{S} \bar{I} - \gamma \bar{I}$$

$$\frac{d\bar{R}}{dt} = \gamma \bar{I}$$

$$\text{Now } \bar{S} + \bar{I} + \bar{R} = 1 \quad \text{and} \quad \bar{S}' + \bar{I}' + \bar{R}' = 0$$

Questions about this model. (dropping bars)

1) Under what conditions will disease die out?

$$\frac{dI}{dt} \Big|_{t=0} < 0$$

$$I' = \beta S I - \gamma I = I(\beta S - \gamma)$$

$$I'(t=0) = I(0)(\beta S(0) - \gamma) < 0$$

If $I(0) \geq 0$, need

$$\beta S(0) - \gamma < 0$$

$$S(0) < \frac{\gamma}{\beta} \quad \text{or} \quad \frac{\beta S(0)}{\gamma} < 1$$

fraction of individuals susceptible? epidemic index < 1

< 1. Small enough so epidemic can't take off

At the onset $S(0) \approx 1$, so

$$\boxed{\frac{\beta}{\gamma} < 1} \quad \text{disease dies out}$$

Intervention will invade. This is a threshold phenomenon and β/R is called R_0 or basic reproduction #.

R_0 = average # of cases arising from 1 primary case in an entirely susceptible pop.

$$\beta = \text{infect rate} = \frac{\text{new infection}}{\text{time period}}$$

$\frac{1}{R_0}$ time in infectious class $\beta \cdot \frac{1}{R_0}$ new infection per person
 R_0 is the most important quantity in disease modeling!
 Ifs not possible for susceptible individuals to escape infection
 i.e. $\lim_{t \rightarrow \infty} S(t) > 0?$

$$\frac{dS}{dt} = \beta \frac{S}{N} (R - S)$$

$$\frac{dS}{dR} = \frac{dS/dt}{dR/dt} = \frac{-\beta S R}{R^2} = -\frac{\beta S}{R}$$

Exercise: solve for R $\frac{dS}{dR} = -R_0 S$
 $S = C e^{-R_0 R(t)}$

Assume $S(0) = S_0$
 $S = S_0 \exp(-R_0 R(t))$
 $\lim_{t \rightarrow \infty} S(t) = S_0 \exp(-R_0 R(\infty))$

there is a positive number S_0 so there is still a fraction of susceptible individuals remaining. Not enough infection to sustain transmission. called epidemic threshold.