

10/25/2022

Some preliminaries

Electricity

Current  $I$  = flow of positive charge.

Voltage/potential  $V$  = energy required to move one unit of charge between 2 pts.

Analogy to flow:  $Q \leftrightarrow I$        $\Delta V \leftrightarrow \Delta P$

Water flows from high to low pressure

Positive charge flows from high to low voltage

Resistor       $\Delta V = IR$

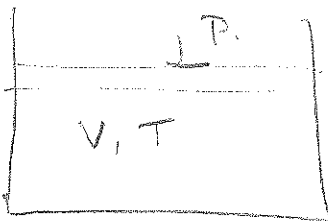


$q$  = elementary charge. Pump around  $I = PQ$

Work

A lot of the topics in this chapter involve work, and strangely the work on an ideal gas

Def Consider a piston-cylinder ideal gas



The work done to go from

state 1  $\rightarrow$  2

$$W = - \int P_{\text{ext}} dV$$

Ex 1 Gas is compressed from 2 L to 1 L at constant  $P = P_0$ . Find the work done.

$$W = -P \int dV = -P_0(1-2) = P_0 \cdot 1$$

Ex 2 Find the work done if  $P_{\text{ext}} = P_{\text{int}}$  and isothermal expansion

$P = \frac{nKT}{V}$  so

$$W = - \int_{V_1}^{V_2} \frac{nKT}{V} dV = -nKT \ln\left(\frac{V_2}{V_1}\right)$$

$V_2 > V_1$  = expansion, work done by system

in terms of concentration,  $c = n/V \rightarrow c_2/c_1 = V_1/V_2$

$$W = nKT \ln\left(\frac{c_2}{c_1}\right)$$

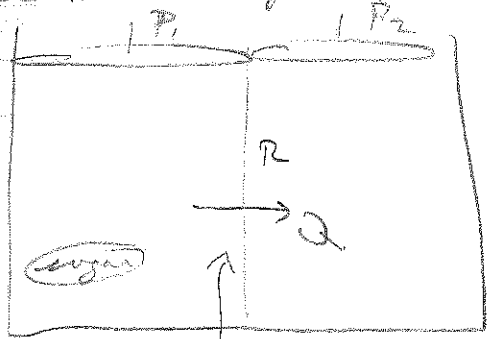
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### Cell volume control and osmosis

The cell contains negatively charged macromolecules like proteins and lipids. These charges are balanced by positively charged ions which are dissolved in water. BUT excess of the + ions inside the cell would normally bring water in by osmosis and burst the cell so how does the cell control its volume? We'll build a model of this.

### Osmotic pressure

Pressure differences drive flows. Osmosis drives flows. Is there some way that we can equate the 2?



Model membrane as resistor with  $P_1 - P_2 = RQ$ .

rigid membrane (permeable to water only)

Now what about when sugar is dissolved in the left compartment? There should be an osmotic flow going towards 1.

IF the sugar were an 'ideal gas' at constant pressure

$$W_{\text{concentration}} = - \int_{P_1}^{P_2} P dV = -c kT (-Q) = Q c kT.$$

Rate of work required to move stuff into compartment

BIG ASSUMPTION: rate of work to concentrate the liquid = rate of work needed to concentrate the gas.

$$\text{So } W_{\text{concentration}} = Q kT c.$$

Where does the work come from? The protein performs work at rate

$$W_{\text{protein}} = (P_1 - P_2) Q.$$

That work is used to change the concentration AND to overcome resistance of the membrane. The latter requires work.

$$W_{\text{membrane}} = R Q^2 \quad (I^2 R)$$

$$\text{Gives us } W_{\text{protein}} = W_{\text{membrane}} + W_{\text{conc.}}$$

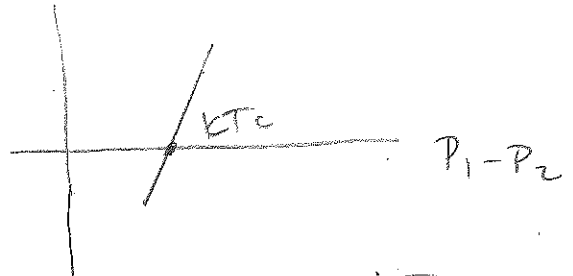
$$(P_1 - P_2) Q = R Q^2 + Q kT c$$

$$P_1 - P_2 = RQ + kTc$$

When  $c = 0$ ,  $Q = (P_1 - P_2) / R$  as expected.

But when  $c \neq 0$ :

$$Q = \frac{P_1 - P_2}{R} - \frac{kTc}{R}$$



Most apply pressure difference  $kTc$  to SDP vs. flow.

In cells, will have multiple species concentrated on interior and exterior. Osmotic pressure is

$$P_1 - P_2 = kT (c_1^{(a)} - c_2^{(a)} + c_1^{(b)} - c_2^{(b)}) = RQ \quad (*)$$

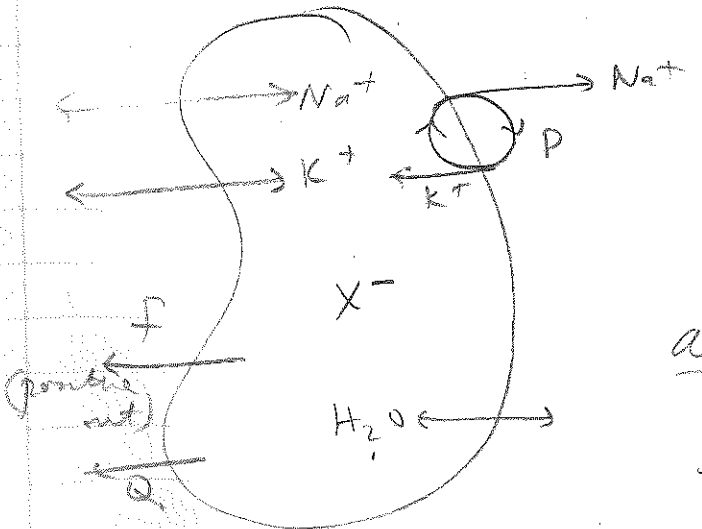
$a, b = \text{species}, 1, 2 = \text{compartments}$

In cell: 1 = inside cell, 2 = outside cell,

$a = Na^+, b = K^+, c = Cl^-$

Simple model of volume control

Ignore electrical effects (ie movement of ions across membrane in response to changes in voltage)



Negative charges denoted by X.  
Concentration  $X/V$ . Cell contains  $Na^+$  and  $K^+$  which can diffuse through membrane AND are actively pumped.

Assume pressure inside and outside the cell is the same so that

$$R_{H_2O} Q = -kT ([Na^+]_i - [Na^+]_o + [K^+]_i - [K^+]_o + \frac{X}{V})$$

which follows from (\*).

Now what about the flux of  $\text{Na}^+$  and  $\text{K}^+$ ?

$$f_{\text{Na}} = \alpha_{\text{Na}} ([\text{Na}^+]_i - [\text{Na}^+]_o) + P \quad \left( \frac{\text{ions}}{\text{time}} \right)$$

$$f_{\text{K}} = \alpha_{\text{K}} ([\text{K}^+]_i - [\text{K}^+]_o) - P$$

$\alpha_{\text{Na}, \text{K}}$  = permeability of cell to  $\text{Na}^+$  and  $\text{K}^+$ . Notice the diffusion seen in eq. while the gradient at steady state, net fluxes are zero.

$\left[ \begin{array}{l} \text{pump out} \\ \text{diffuse in} \end{array} \right] [\text{Na}^+]_i - [\text{Na}^+]_o = -\frac{P}{\alpha_{\text{Na}}} \quad \left. \vphantom{\left[ \begin{array}{l} \text{pump out} \\ \text{diffuse in} \end{array} \right]} \right\} \text{concentration differences produced by pump}$

$\left[ \begin{array}{l} \text{pump in} \\ \text{diffuse out} \end{array} \right] [\text{K}^+]_i - [\text{K}^+]_o = \frac{P}{\alpha_{\text{K}}}$

$\left[ \begin{array}{l} \text{ionic diff.} \\ \text{neg changes} \end{array} \right] [\text{Na}^+]_i - [\text{Na}^+]_o + [\text{K}^+]_i - [\text{K}^+]_o = -\frac{X}{V}$

Exercise: this is 3 eqns. for unknowns  $V$ ,  $[\text{Na}^+]_i$ ,  $[\text{K}^+]_i$ .

Solve for  $V$ :

$$-\frac{P}{\alpha_{\text{Na}}} + \frac{P}{\alpha_{\text{K}}} = -\frac{X}{V} \rightarrow \frac{X}{V} = \frac{P \alpha_{\text{K}} - P \alpha_{\text{Na}}}{\alpha_{\text{K}} \alpha_{\text{Na}}}$$

$$\rightarrow V = X \frac{\alpha_{\text{K}} \alpha_{\text{Na}}}{P(\alpha_{\text{K}} - \alpha_{\text{Na}})}$$

### Implications

- 1) Steady state volume only when  $\alpha_{\text{K}} > \alpha_{\text{Na}}$ .  
Cell maintains SS volume by pumping Na out and K in to produce concentration differences. These concentration differences must be larger for Na than K to balance the negative charge inside. To accomplish this  $\alpha_{\text{Na}} < \alpha_{\text{K}}$ .
- 2)  $V \propto 1/p$ . As pump rate decreases, cell swells up and lysis as we discussed at the beginning.
- 3)  $V \propto X$ . The amount of macromolecules, etc. is a proxy for the size of the cell.