HW 9 Solution

1\. (a) $\frac{dS}{dt} = -\beta S I + \alpha I^2$

$\frac{dI}{dt} = \beta S I - \alpha I^2$

(b) $S = 1 - I$,

$\frac{dI}{dt} = \beta(1-I)I - \alpha I^2$

$= \beta I - \beta I^2 - \alpha I^2$

$= \beta I - (\alpha+\beta) I^2$

$= I(\beta - (\alpha+\beta) I)$

(c) $I = 0$ (DFE) and $I = \frac{\beta}{\alpha+\beta}$ (endemic eq.)

$f(I) = \beta I - (\alpha+\beta) I^2$

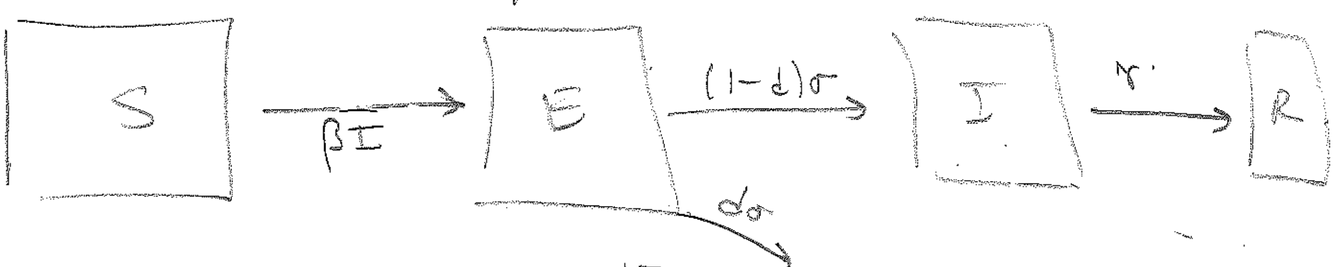
$f'(I) = \beta - 2(\alpha+\beta) I$

if $I = 0$, $f'(I) = \beta > 0$ so unstable DFE

if $I = \frac{\beta}{\alpha+\beta}$, $f'(I) = -\beta < 0$ so stable endemic eq.

2\. (a) Birth and death can be ignored – timescale is slower.

(b)



$\frac{dS}{dt} = -\beta S I$

$\frac{dE}{dt} = \beta S I - \sigma E$

$\frac{dI}{dt} = (1-d)\sigma E - \gamma I$

$\frac{dR}{dt} = \gamma I$

(c) $\frac{d(S+E+I+R)}{dt} = -\sigma E + (1-d)\sigma E$

$= -d\sigma E$

Individuals dying

(d) $\frac{dI}{dt} = 0 \rightarrow I = 0$. No endemic eq. because there are no births and deaths to replenish susceptibles.