MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 9
Due: Tuesday 11/22 3:30 PM

1) Let's consider an SI model like we did in class, but with some twists. Specifically, suppose that infected people pass back into the susceptible class by contacting each other (with rate α), in exactly the same way that susceptibles pass into the infectious class by contacting infectious people. Assume no births and deaths here.

(a) Using this model, write the evolution equations for S and I. [2 pts]

(b) Substitute $S = 1 - I$, to find an ODE for I only. [1 pt]

(c) Find the steady states of this ODE and determine their stability. Give a physical explanation for the steady states.[4 pts]

2) Consider a disease where susceptible individuals acquire the infection, but spend some duration of time $1/\sigma$ incubating the infection prior to being exposed. After incubating the infection, they either die immediately (with probability d) or become infectious. After becoming infectious, individuals recover.

(a) The dynamics of this disease play out over a span of a few weeks. What does this mean about accounting for births and deaths in the model? [1 pt]

(b) Divide the population into four compartments (S, E, I, and R), and write the ODEs governing the changes in those populations over time. [4 pts]

(c) Use your answer to (b) to evaluate

$$\frac{d(S + E + I + R)}{dt}$$

Explain your answer physically. Is the population conserved? Why or why not? [1 pt]

(d) What are the equilibrium points? Is there an endemic equilibrium? Why or why not? [2 pts]