MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 7
Due: Tuesday 11/01 3:30 PM

1) In class, we discussed how the reversible work done on a gas is given by $W = -\int PdV$, and we derived the work done for isothermal expansion of an ideal gas. Suppose that you are given an alternate equation of state (van der Waals equation of state) where

$$P = \frac{RT}{V-b} - \frac{a}{V^2}, \tag{1}$$

where R, a, and b are constants. Determine the work done by isothermal compression of an ideal gas between V_1 and $V_2 < V_1$. [3 pts]

2) Consider a membrane immersed in a hypertonic salt solution. The external solution has $[\mathrm{Na}]^{\mathrm{ext}} = 140$ mEq/L. The internal solution has unknown concentrations $[\mathrm{Na}]^{\mathrm{int}}$, but there is a known charge X of all the negative ions. The membrane has voltage v and volume V, and sodium is pumped *in* with rate p ions per unit time.

(a) Find an expression for the osmotic flux of water out of the cell Q, assuming R_H is the resistance of the membrane to water outflow. [1 pt]

(b) If the membrane has voltage v, write an expression for the total current of Na if the membrane conductance is g_{Na}. [1 pt]

(c) If the interior of the membrane contains only sodium and the negative charges, and it is electrically neutral, find $[\mathrm{Na}]^{\mathrm{int}}$ in terms of X and V [1 pt].

(d) Assuming the membrane is at steady state, solve for v, V, and $[\mathrm{Na}]^{\mathrm{int}}$. Your answer for $[\mathrm{Na}]^{\mathrm{int}}$ should be a number. [3 pts]

(e) Sketch (no need for a computer plot) a graph of v as a function of the pump rate p. Indicate the slope and y-intercept. [1 pt]

(f) When $p = 0$, is the voltage positive or negative? Explain why. [1 pt]

(g) Explain what happens to the membrane voltage v as p increases and why this is a consequence of steady state. [1 pt]