MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 6
Due: Tuesday 10/25 3:30 PM

1) In class, we assumed that all cross bridges attach at $x = A$, where $A > 0$, and detach at a constant rate independent of x. Let's consider a different model in this homework. In particular, we will assume that attachment occurs at an arbitrary value of x, and detachment occurs at a rate that increases with strain. As in class, you will use n_0 as the number of CBs per half sarcomere and

$$U = \int_{-\infty}^{\infty} u(x)\, dx$$

as the total fraction of attached bridges.

(a) In particular, suppose that attachment of the CBs occurs at an arbitrary x in $[-A, A]$ with constant rate (per unit length) α_0. Write an integral expression that gives the total rate (number/time) of attachment pet unit time on an interval $[x_0, A]$ with $-A < x_0 < A$. [1 pt]

(b) Now assume that the rate of detachment for an individual bridge is $\beta e^{x/A}$. Write an integral expression that gives the total rate (number/time) of detachment per unit time on an interval $[x_0, A]$ with $-A < x_0 < A$. [1 pt]

(c) Assuming that v is the sliding speed, give the rate (number/time) at which bridges slide past $x = x_0$. [1 pt]

(d) Use your answers for (a)–(c) to write an ODE (differentiate the integral equations) governing $u(x)$. Do not solve the ODE. [2 pt]

For the rest of this homework, set $v = 0$. This means that cross bridges attach and detach only in the interval $-A < x < A$, and therefore that

$$U = \int_{-A}^{A} u(x)\, dx$$

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(e) Now solve the equation in (d) to obtain an expression for $u(x)$ when $v = 0$. Note that you will need to solve for U to get full credit. [3 pts]

(f) Assume that each CB has the force-extension relationship $p(x) = p_0\left(e^{kx/A} - 1\right)$. Determine the total force P when $v = 0$. [2 pts]

(g) What is the minimum positive integer k that gives a positive force P? [1 pt]