HW 5 Solutions

1. $\frac{dQ}{dx} = -f_{H_2O}$

(a) On $4 \le x \le 5$, $f_{H_2O} = (x-4)(5-x)$

$= +x^2 - 9x + 20$

$Q(x) = \int +x^2 - 9x + 20 \, dx$

$+\frac{x^3}{3} - \frac{9x^2}{2} + 20x + C$

at $x = 4$, $Q = 2$.

$Q(4) = +\frac{64}{3} - \frac{9 \cdot 16}{2} + 80 + C = 2$

$= +\frac{88}{3} + C = 2 \rightarrow C = -\frac{82}{3}$

$$Q(x) = \begin{cases} \frac{x^3}{3} - \frac{9x^2}{2} + 20x - \frac{82}{3} & 4 \le x \le 5 \\ 1.8\overline{3} = \frac{11}{6} \text{ L/min} & x \ge 5 \end{cases} \qquad x \le 4.$$

(b) 1.83 L/min (the same as at $x = 5$, which is obtained by evaluating $\frac{x^3}{3} - \frac{9x^2}{2} + 20x - \frac{82}{3}$ at $x = 5$.

(c) 1/6 L/min

(d) $\frac{40 \text{ L}}{1/6 \text{ L/min}} = 240 \text{ min} = 4 \text{ hrs.}$

(e) $Q(0)\,c(0) = Q(10)\,c(10)$

$2\,c(0) = \frac{11}{6}\,c(10) \rightarrow$ $\boxed{c(10) = \frac{12\,c(0)}{11}}$

(f) $35 = \frac{12}{11}\,c(0) \rightarrow c(0) = \frac{385}{12} \approx$ $\boxed{32.1 \text{ g/L}}$

(g) Amt Input = $Q(0)\,c(0)$

Amt Output = Input $- f^*_{Na} L = Q(0)c(0) - f^*_{Na} L = 80 - 10 f^*_{Na}$

Output conc. = $\frac{\text{Amt}}{Q(10)} = \frac{80 - 10 f^*_{Na} \text{ g/min}}{11/6 \text{ L/min}} = 35$

$\rightarrow$ $\boxed{f^*_{Na} = 1.583 \text{ g/m·min}}$