

MATH/BIOL 255: Mathematics in Medicine and Biology

Homework 5

Due: Tuesday 10/18 3:30 PM

1) You are using a 10 meter long garden hose to fill up a salt-water swimming pool in your backyard. The inflow of the hose is 2 L/min. There is, however, a small 1 meter section beginning 4 meters from the start of the hose which has a leak in it. The leak gets wider along the section, then narrows again, so that the flux per unit length of water leaking out of the hose is

$$f_{\text{H}_2\text{O}}(x) = \begin{cases} 0 & x \leq 4 \\ (x - 4)(5 - x) & 4 \leq x \leq 5 \\ 0 & x \geq 5 \end{cases}$$

- (a) Solve an ODE for the flow rate $Q(x)$ of the water in the hose. [3 pts]
- (b) How fast is water entering the pool? [1 pt]
- (c) What is the total amount of water leaking out of the hose per time? [1 pt]
- (d) You have a 10 gallon (1 gallon \approx 4 L) bucket that collects the leaking water. How often will you have to change the bucket to prevent it overflowing? [1 pt]

Now let's consider the salt-water content of the pool. The goal is for the concentration to be equal to that of sea water (35 g/L).

- (e) Suppose that the holes in the hose act as a filter, so that no salt passes through them, only water. In that case, determine the concentration of salt leaving the hose as a function of the input concentration $c(0)$. [2 pts]
- (f) What should the input concentration $c(0)$ be to get the correct concentration in the pool under these circumstances? [1 pt]
- (g) Now suppose you are given an input concentration $c(0) = 40$ g/L, and you want to pump sodium out of the hose at constant rate f_{Na}^* (per unit length) so that the pool gets a concentration of $c(10) = 35$ g/L. What is the rate of pumping (per meter of hose)? [3 pts]