Homework #4 Solution

1. (Text problem 1.15)

$R_d = 0 \rightarrow P_{sa} = P_{pa} = P_a$

$K_R = K_L = K$

f.o. is CLOSED $\rightarrow$ $Q_f = 0$ $\quad P_{sv} = P_{pv}$. (need to check)

If $Q_f = 0 \rightarrow$

$$Q_p = Q_L \quad (1)$$
$$Q_s = Q_R \quad (2)$$
$$Q_d + Q_L = Q_s \quad (3)$$
$$Q_R = Q_p + Q_d \text{ (redundant)}$$

$$Q_R = K P_{sv} \quad (4)$$
$$Q_L = K P_{pv} \quad (5)$$
$$Q_p = (P_a - P_{pv})/R_p \quad (6)$$
$$Q_s = (P_a - P_{sv})/R_s \quad (7)$$

7 eqns in 8 unknowns ($Q_p, Q_L, Q_s, Q_R, Q_d, P_a, P_{pv}, P_{sv}$)

Solve for everything in terms of P_{pv}:

Combine equations to get

$$Q_s = K P_{sv} = (P_a - P_{sv})/R_s \Rightarrow P_a = K R_s P_{sv} + P_{sv}$$
$$Q_p = K P_{pv} = (P_a - P_{pv})/R_p \quad P_a = K R_p P_{pv} + P_{pv}$$

$$\rightarrow P_{sv}(1 + K R_s) = P_{pv}(1 + K R_p)$$

$$\frac{P_{sv}}{P_{pv}} = \frac{1 + K R_p}{1 + K R_s} < 1 \quad \text{if} \quad R_s > R_p$$

so $P_{sv} \leq P_{pv}$ indeed f.o. is CLOSED.

Solving for Q_d:

$$Q_d = Q_s - Q_L = K P_{sv} - K P_{pv}$$
$$= K(P_{sv} - P_{pv})$$
$$= K P_{pv}\left(\frac{P_{sv}}{P_{pv}} - 1\right)$$
$$= K P_{pv}\left(\frac{1 + K R_p}{1 + K R_s} - \frac{(1 + K R_s)}{1 + K R_s}\right)$$
$$= \frac{K^2 P_{pv}(R_p - R_s)}{1 + K R_s} < 0$$