Homework 3 Solution

1. (a) $V_{pa} = C_{pa} P_{pa}$ $V_{pv} = C_{pv} P_{pv}$

$V_{sa} = C_{sa} P_{sa}$ $V_{sv} = C_{sv} P_{sv}$

(b) $Q R_p = P_{pa} - P_{pv}$

$Q R_s = P_{sa} - P_{sv}$.

(c) $V_0 = V_{pa} + V_{sa} + V_{pv} + V_{sv}$ (total volume constant)

$P_{pa} = P_{sv}$! (Because of missing right heart)

$Q = K_L P_{pv}$. (left heart only!).

(d) Pressures in terms of Q

$$P_{pv} = Q/K_L$$
$$P_{pa} = Q(R_p + 1/K_L) = Q R_p + P_{pv}$$
$$P_{sv} = P_{pa} = Q(R_p + 1/K_L)$$
$$P_{sa} = Q R_s + P_{sv}$$
$$= Q(R_s + R_p + 1/K_L).$$

So, with compliance relations

$$V_0 = V_{pa} + V_{sa} + V_{pv} + V_{sv}.$$
$$V_0 = Q\left[C_{pa}(R_p + 1/K_L) + C_{sa}(R_s + R_p + 1/K_L) + C_{pv}/K_L + C_{sv}(R_p + 1/K_L)\right].$$

Can already see that when $R_p + 1/K_L = K_R$, we'll get equivalent flows

$$\boxed{Q = \frac{V_0}{C_{pa}(R_p + 1/K_L) + C_{sa}(R_s + R_p + 1/K_L) + \frac{C_{pv}}{K_L} + C_{sv}(R_p + 1/K_L)}}$$

(e) Using table 1.2:

$$Q = \frac{5}{\left(0.006667\left(1.79 + \frac{1}{1.12}\right) + 0.01\left(17.5 + 1.79 + \frac{1}{1.12}\right) + \frac{0.08}{1.12} + 1.75\left(1.79 + 1/1.12\right)\right)}$$

$\boxed{Q = 1 \text{ L/min}}$! Incredibly weak!

Previous system had 5.6 L/min

(f) solve for K_L s.t. $Q = 5.6$ L/min

$$\frac{5}{5.6} = 0.00667\left(1.79 + \frac{1}{K_L}\right) + 0.01\left(17.5 + 1.79 + \frac{1}{K_L}\right)$$

$$+ \frac{0.08}{K_L} + 1.75\left(1.79 + \frac{1}{K_L}\right)$$

$$\frac{5}{5.6} - (0.00667)(1.79) - (0.01)(17.5 + 1.79) - (1.75)(1.79)$$

$$= \frac{1}{K_L}\left(0.08 + 1.75 + 0.00667 + 0.01\right)$$

$$\frac{1}{K_L} = -1.32 \rightarrow K_L = -0.75$$

NOT possible!

2. (a) $$\frac{dP}{dz} = \frac{Pg}{RT}$$

$$\frac{dP}{P} = \frac{g}{RT}\,dz \qquad \ln P = \frac{gz}{RT} + c$$

$$\boxed{P = P_{atm} e^{-gz/RT}}$$

(b) $P_{O_2} = 0.2 P_{atm} e^{-gz/RT}$

(c) P_{O_2} in air = P_{O_2} in gas

$$C = \sigma P_{O_2} = 0.20 P_{atm} e^{-gz/RT}$$

(d) $M = Q[O_2]_a$ at min.

$$Q = \frac{M}{[O_2]_a} = \frac{M_0}{(1 + e^{-t/H})(0.20 P_{atm} e^{-gz/RT})}$$

$$\boxed{Q(z) = e^{gz/RT}\,\frac{M_0}{0.20 P_{atm}} \cdot \frac{1}{1 + e^{-t/H}}}$$

(e) $$\boxed{F(z) = \frac{Q(z)}{V_{stroke}}}$$