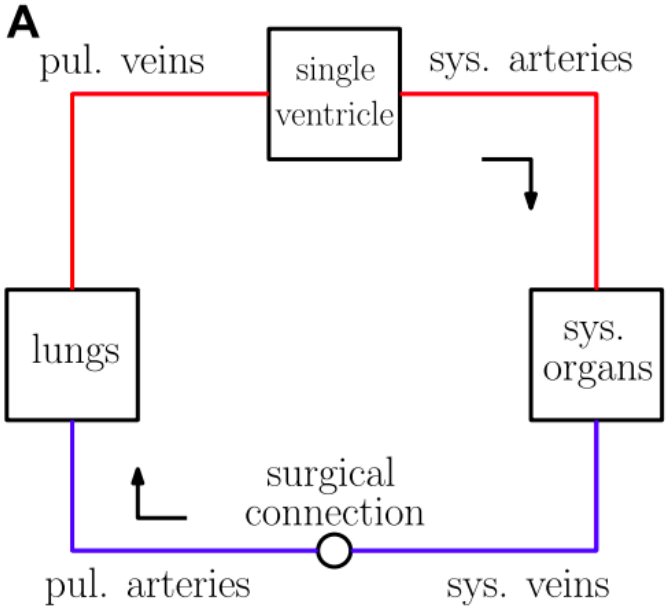
MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 3
Due: Tuesday 09/27 3:30 PM

1) Consider a patient with heart disease that only has a single working heart chamber. The diagram below shows the schematic of this patient's circulation.



As in class, the model unknowns are the four pressures $P_{\text{pa}}, P_{\text{pv}}, P_{\text{sa}}, P_{\text{sv}}$, the four corresponding volumes, and the flow rate Q. Assuming the total volume of blood is constant at $V_0 = 5$ L, you will have 9 equations in 9 unknowns.

(a) Write the relationship between the volume and pressure in the veins and arteries (4 equations). [1 pt]

(b) Write the relationship between the flow rate Q and the pressure drop across the systemic organs and lungs (2 equations). [1 pt]

(c) What are the three remaining equations? Describe their meaning physically and explain how they differ (if at all) from the model derived in class. [3 pts]

(d) Solve the model to obtain the cardiac output Q. [2 pts]

(e) Using the "normal" parameters given in text Table 1.2, compare the cardiac output to a person with a normal heart. Treat the single ventricle as the left heart. [1 pt]

(f) Is it possible for the single ventricle to pump enough blood to restore normal cardiac output? Hint: what would the pump coefficient of the single ventricle have to be to obtain $Q = 5.6$ L/min? [2 pts]

2) Remember (or learn) that the pressure of the air P_{atm} changes with the height above sea level via the equation

$$\frac{dP}{dz} = -\rho_a g \tag{1}$$

where g is the acceleration due to gravity and the density of air is given by the ideal gas law as

$$\rho_a(z) = \frac{P(z)}{RT}, \tag{2}$$

(R is the gas constant and T the temperature).

(a) Solve for $P(z)$ if the pressure at sea level is P_{atm}. [2 pts]

(b) If air is 20% oxygen, find the partial pressure of oxygen in air. [1 pt]

(c) Suppose that the arterial blood is in equilibrium with the inspired air, and that oxygen forms a simple solution in blood with solubility σ. What is the concentration of oxygen in the arterial blood? [2 pts]

(d) Suppose that a hiker is climbing at a constant speed from $z = 0$ to $z = H$ meters. In addition, the metabolic rate at which the hiker's muscles burn oxygen increases with z as

$$M(z) = \frac{M_0}{1 + e^{-z/H}}. \tag{3}$$

Find the minimum flow rate $Q(z)$ required to sustain this metabolic rate. [2 pts]

(e) Suppose that the hiker's stroke volume V_{stroke} is relatively constant during the hike. Find an expression for how the heart rate $F(z)$ changes with height. [1 pt]