MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 2
Due: Tuesday 09/20 3:30 PM

1) Circulation times (variant of textbook problem 1.1).

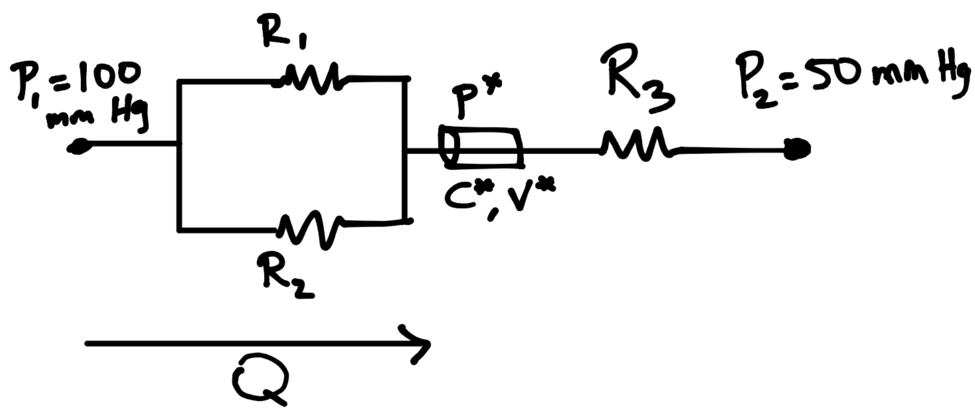
(a) Given that the total volume of blood in the circulation is $V_0 = 5$ L and the cardiac output is $Q_0 = 5.6$ L/min, calculate the time it takes for a "typical" red blood cell to go once around the circulation. This is called the *circulation time.* [2 pts]

(b) Consider a more realistic model in which the circulation consists of n parallel loops. Let the volumes occupied by these loops be $V_1, V_2, \ldots V_n$, and their blood flows be $Q_1, \ldots Q_n$. Define $V_0 = \sum_i V_i$ and $Q_0 = \sum_i Q_i$. Show that the weighted average circulation time of the parallel loops

$$\langle T_c \rangle = \frac{\sum_i T_i Q_i}{\sum_i Q_i}$$

is equal to the circulation time of a single circulatory system with blood volume V_0 and cardiac output Q_0. [2 pts]

2) Consider a network of blood vessels moving fluid from high to low pressure as shown below. There are two resistance vessels in parallel, followed by a compliance vessel, and then a third resistance vessel.



(a) Suppose only 2/3 of the total blood volume can flow through the second resistance vessel (the one with resistance R_2). If $R_1 = 6$ mm Hg/(L/min), what is the minimum value of R_2? [2 pts]

(b) Suppose R_2 takes its minimum value. What is P^* if the total volume of blood flow is 4 L/min? [2 pts]

(c) Now consider the compliance vessel. Suppose it has dead volume $V_d^* = 5$ L and the vessel is at its maximum volume of $1.5V_d^*$. What is C^*? [2 pts]

(d) What is the resistance R_3 under these circumstances? [2 pts]

(e) How much work (per unit time) is required to complete the circuit (i.e., pump the blood from the final node to the first node)? [2 pts]