HW 11 Solutions

$$u_t = au + (u-1)v \quad := f(u,v)$$
$$v_t = -bv + (1-v)u \quad := g(u,v).$$

(a) Without competition, $u = u_0 e^{at}$ (exponential growth), while $v = V_0 e^{-bt}$ (exponential decay)

(b)

	$u<1$	$u>1$
$v<1$	$u\downarrow$ $v\uparrow$ v PRED, u PREY	$u\uparrow$ $v\uparrow$ SYMBIOSIS
$v>1$	$u\downarrow$, $v\downarrow$ COMPETITION	$u\uparrow$ $v\downarrow$ u PRED, v PREY

(c) $(0,0)$ is always SS. #1.

Second steady state

$$au + (u-1)v = 0 \rightarrow v = -\frac{au}{u-1}$$
$$-bv + (1-v)u = 0$$
$$\hookrightarrow \frac{abu}{u-1} + \left(1 + \frac{au}{u-1}\right)u = 0$$
$$ab\cancel{u} + \cancel{u}(u-1) + a\cancel{u}^2 = 0$$
$$ab + (u-1) + au = 0$$
$$\boxed{u^* = \frac{1-ab}{1+a}}$$

$$u-1 = \frac{1-ab}{1+a} - \frac{1+a}{1+a} = -\frac{ab-a}{1+a} \qquad (u-1)^{-1} = -\left(\frac{1+a}{ab+a}\right)$$
$$\longrightarrow v = +a\left(\frac{1-ab}{\cancel{1+a}}\right)\left(\frac{\cancel{1+a}}{ab+a}\right) = \boxed{\frac{a(1-ab)}{ab+a} = v^*}$$

(d) $ab < 1$

(e) $a = 1/2,\ b = 2/3 \rightarrow u^* = 4/9 \qquad v^* = 2/5$

$$J = \begin{pmatrix} f_u & f_v \\ g_u & g_v \end{pmatrix}\Bigg|_{u^*,v^*} = \begin{pmatrix} a+v & u-1 \\ 1-v & -b-u \end{pmatrix}$$

Setting $a = 1/2$, $b = 2/3$, $u = 4/9$, $v = 2/5$ get

$$J = \begin{pmatrix} 9/10 & -5/9 \\ 3/5 & -10/9 \end{pmatrix} \qquad \det J = -2/3 < 0$$

UNSTABLE

(e) at 0,

$$J = \begin{pmatrix} a & -1 \\ 1 & -b \end{pmatrix} = \begin{pmatrix} 1/2 & -1 \\ 1 & -2/3 \end{pmatrix}$$

$$\det J = -\frac{1}{3} + 1 = 2/3$$

$$\mathrm{tr}\, J = \frac{1}{2} - \frac{2}{3} < 0.$$

2 negative eigenvalues!

So 0 is the only stable pt.

(f) It goes to 0 which is the only stable point.

HW 11: Due 12/14 11:59 PM

OH 14, 1-3

R 1-3