MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 11
Due: Wednesday 12/14 11:59 PM

1) Consider a pair of species (u, v) which evolve according to the set of equations

$$\frac{du}{dt} = au + (u-1)v$$
$$\frac{dv}{dt} = -bv + (1-v)u,$$

where a and b are positive constants.

(a) What happens to species u if there is no competition (if $v = 0$)? Likewise, what happens to species v if there is no competition (if $u = 0$)? [2 pts]

(b) Notice that the interaction terms change their sign if based on if u and v are greater or smaller than 1. With that in mind, fill in the table below with the words "competition," "symbiosis," or "predation" for the different regimes. If predation, identify the predator and prey. [4 pts]

	$u < 1$	$u > 1$
$v < 1$		
$v > 1$		

(c) Find the steady states (there are two) for this system. [3 pts]

(d) When is the steady state physical? [1 pt]

(e) Setting $a = 1/2$ and $b = 2/3$, compute the Jacobian matrix for this system and use it to determine the stability of *both* steady states. [4 pts]

(f) Where does the population tend to in the long term? [1 pt]