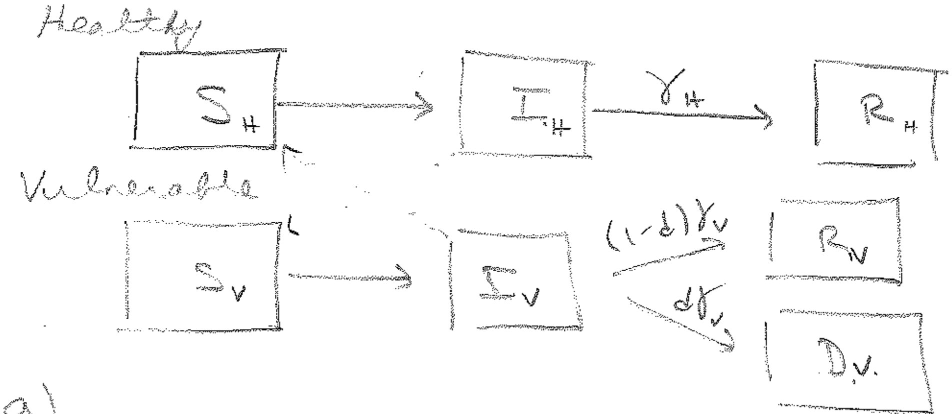
HW10 Solutions

1\.



(a)

$$\frac{dS_H}{dt} = -\beta_{HH} S_H I_H - \beta_{HV} S_H I_V.$$

$$\frac{dI_H}{dt} = \beta_{HH} S_H I_H + \beta_{HV} S_H I_V - \gamma_H I_H$$

$$\frac{dR_H}{dt} = \gamma_H I_H$$

$$\frac{dS_V}{dt} = -\beta_{VV} S_V I_V - \beta_{VH} S_V I_H$$

$$\frac{dI_V}{dt} = \beta_{VV} S_V I_V + \beta_{VH} S_V I_H - \gamma_V I_V.$$

$$\frac{dR_V}{dt} = \gamma_V I_V (1-d)$$

$$\frac{dD_V}{dt} = d\gamma_V I_V.$$

(b) Keep $\frac{\beta_{HH}}{\gamma_H}$ constant (disease has same R_0 in the healthy people).

(β_{HH}, γ_H)	% vulnerable died
$(1/10, 1/50)$	20%
$(1, 1/5)$	19.8%
$(10, 2)$	7.2%

The epidemic spreads faster among healthy people as (β_{HH}, γ_H) increase. This means they have less time to infect the vulnerable people.

(c)

$N(0)$	0.01	1.2	1.8	2.2	3.5	5
$N(\infty)$	1	1	1	4	4	4.

(a) By the IVT, there is one eq pt between 0.5 and 1.5 one between 1.5 and 2.5, and another between 2.5 and 4.2. So there are 3 eq pt at least.

(b) $F(N) = aN^3 + bN^2 + cN + d$

$$\begin{pmatrix} 2.625 \\ -0.625 \\ 1.125 \\ -1.408 \end{pmatrix} = \begin{pmatrix} 0.5^3 & 0.5^2 & 0.5 & 1 \\ 1.5^3 & 1.5^2 & 1.5 & 1 \\ 2.5^3 & 2.5^2 & 2.5 & 1 \\ 4.2^3 & 4.2^2 & 4.2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\Longrightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \\ -14 \\ 8 \end{pmatrix}$$

(c) $F(N) = -N^3 + 7N^2 - 14N + 8$

1 is a root

$$\begin{array}{r|l} & -N^2 + 6N - 8 \\ \hline N-1 & -N^3 + 7N^2 - 14N + 8 \\ & -(-N^3 + N^2) \\ \hline & 6N^2 - 14N \\ & -(6N^2 - 6N) \\ \hline & -8N + 8 \\ & -(-8N + 8) \\ \hline & 0 \end{array}$$

$-(N-1)(N^2 - 6N + 8)$

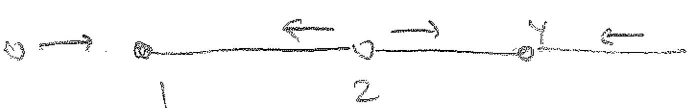
$-(N-1)(N-4)(N-2)$ $\boxed{N = 1, 2, 4}$

(d) $F'(N) = -3N^2 + 14N - 14$

$F'(1) = -3 + 14 - 14 < 0$ STABLE

$F'(2) = -12 + 28 - 14 > 0$ UNSTABLE

$F'(4) = -48 + 56 - 14 < 0$ STABLE



Population has 2 stable states, 1, and 4. If it goes above 2, the steady state of 4 is reached.