

Mathematics and the Glass Bead Game

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The mood at the International Congress of Mathematicians in Philadelphia last summer was, for many, both anxious and somber. Even witnessing the pomp of yet another four mathematicians crowned with our profession's most visible and esteemed award did not seem to raise spirits much (the West Philadelphia Brass Band's klezmer music, played at the opening ceremony, notwithstanding). At least in my case, every single encounter I had over the course of the Congress inevitably led to a discussion of the Sword of Damocles now hanging over our collective necks. That Sword is electronic and probably plastered with ads for the twee named LLMs that mirror our lives: Gemini, Muse, Claude, Fable, etc. Needless to say, Jacob Tsimerman, one of the four Fields medalists announced last summer, bolstered our fears by announcing his departure from academic mathematics to a presumably more favorable position as a safety researcher at OpenAI, one of the frontier labs.

Our anxieties, especially among younger mathematicians like myself, are simple to state in the form of questions: Have I wasted my investment of time, effort, and love in an activity soon to be automated by AI? Will it still be possible to attain an academic research position in mathematics? If mathematics can be automated, is there any human activity that cannot? Terence Tao, in his public lecture at the Congress, attempted to address these questions. For me, his response to how we should approach the use of LLMs and the future of mathematics was a bit unsatisfying. He proposed that if in the future AI is able to solve many of the problems we consider difficult today - and recent evidence, such as the res-

olution of one of the Millennium Problems, suggests this is a likely hypothesis - we mathematicians should relegate ourselves to the process of "canonicalizing" mathematics. In other words, mathematicians will be demoted to determining which of the infinitely many propositions autonomously proven by AI systems we should include in textbooks for the future generation of (presumably) human students. Besides how simply boring this role appears, a more practical (and in my case, as a recent Ph.D. graduate, selfish) concern also arises: Who determines what is important and interesting? Who determines what should be "canonicalized"? After Tao's lecture, I immediately had an unsavory image in my mind of the future practice of mathematics as a version of the art world: billionaire aesthetes paying for tokens on top-of-the-line AI systems, proving new results and collecting theorems to pair with their Rembrandts and Picassos.

As it happened, I was reading a novel by the German-Swiss writer Hermann Hesse during the Congress, and the book served both as a comfort and as inspiration for this note, a modest response to Tao's public lecture and the attitude of pessimistic discontent I encountered in Philadelphia. The novel takes place during an indeterminate period in the future and its title, "The Glass Bead Game", refers to an activity that serves as both intellectual sport and obsessive passion for the few remaining scholars in society. While the rest of the world continues to develop in the active pursuit of business, politics, and war, a few intellectuals retreat to "Castalia", a mountainous refuge whose inhabitants are devoted to the study of the "Game". The rules of the Game are described in the novel as a "secret language drawing upon... mathematics and music" that is "capa-

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ble of expressing and establishing interrelationships between the content and conclusions of nearly all scholarly disciplines”.¹ In a striking parallel with the way LLMs are trained and used today, the description continues by stating that “all the insights, noble thoughts, and works of art that the human race has produced in its creative eras” have been “reduced to concepts and converted into intellectual property” for use by players of the Game. Thus, we shouldn’t mistake Castalia for some idealized university or glamorous think-tank. The Game so devotedly studied by the scholars of Castalia is nothing but an aesthetic distillation of all the ideas that have come before (a “good” player is described as one adept at pleasantly arranging a collection of hieroglyphics, each of which represents some platonic ideal of music, mathematics, or art). Although there are many able mathematicians and musicians in Castalia, no one bothers to prove a *new* mathematical theorem or to write a *new* piano concerto. Even writing a poem is considered bad taste, a mistaken use of one’s time.

Imagine a world where the strongest AI models act as pay-to-play oracles: If you have any reasonable mathematical question and a few thousand dollars, you can determine an answer (and it’s even Lean formalized!). In such a world Tao suggests that the mathematician serves the role of guide and chronicler of the AI’s autonomous ventures into the far reaches of mathematical knowledge, but it is not too great a leap of imagination that such a role would devolve into the world of sterile Castalia, full of overbred players of a purely aesthetic intellectual recreation. The challenge of mathematics seems to disappear, and the zest of creation and discovery is eliminated. Moreover, if AI models can “solve” mathematics in this way, there is probably no bound to their capabilities, so what remains to be done by the humans who created them?

I would like to argue that the dilemma described above is an artificial by-product of the present culture of mathematics, something also hinted at by Tao in his lecture. For many years mathematicians have viewed our field as akin to a sporting arena of the

wits. Intellectual mettle is tested one against another in trying to solve a series of ever more difficult problems, accumulating accolades like the Fields Medal, publishing a paper in a “top five” journal, or simply finding a tenure-track job. These are the rewards for the victorious. For many mathematicians, the field has become exactly like the “Glass Bead Game,” an abstruse intellectual activity appreciated by few and giving pleasure to even fewer. Certainly, mathematicians have much to fear from AI models if they ruin the fun of our game and solve all our problems for us!

The process of making mathematics a Glass Bead Game of manipulating logical propositions began with David Hilbert and now ends with the computer.² Hilbert’s program at the start of the previous century bears a striking resemblance to the proposed use of our current AI models. If mathematics amounts to manipulating strings of information according to axiomatic rules, then of course the computer, the string manipulator par excellence, will outperform and possibly replace us. Anyone who has dabbled with Olympiad math in high school remembers the *modus operandi* of that genre of problem solving: memorize a thousand techniques and apply the correct technique to the problem as quickly as possible. It goes without saying that a Large Language Model (LLM) can do this better than a human (recent LLMs can easily solve the IMO problems and we note that even in the 1990s chess masters were defeated by well-trained computers). What we should take from the ability of the AI models is *not* that the human input to mathematics is replaceable but rather that it was not the essentially human part of mathematics to begin with. The top mathematicians of the eighteenth century were known to devote a significant amount of

¹All quotations from the novel are taken from the translation by Richard and Clara Winston [Hes70].

²While it is unclear whether Hesse was aware of Hilbert’s program to formalize mathematics, there is evidence that Hesse engaged with the mathematical developments of his time. In his 1932 review of Hermann Broch’s “Die Schlafwandler,” Hesse describes the interpolated essay on the disintegration of values, which includes a discussion on the foundations of mathematics, as the work’s central intellectual component. At the very least, this shows that Hesse had opportunity to encounter contemporaneous mathematics and its relevant concerns. Hesse’s review [Hes32] is quoted in [Sch19, p. 153, n. 70]; for Broch’s mathematical context, see [Eng18, chap. 2, pp. 59–92].

their working day to simple arithmetic, something we delegate to pocket calculators today. Did that eliminate anything significant from the human experience, wonder, and joy of mathematics?

Let us propose an alternative, more optimistic viewpoint of the future of mathematics: a viewpoint that ultimately grants Poincaré the final victory over Hilbert nearly a century after their battle seemed to have played out.³ We should embrace the idea of mathematics as a natural science, a field of human thought where intuition and discovery prevail over mere symbolic manipulations and clever game-playing. After all, there is ample evidence (the myriad applications to the physical and social sciences) that mathematics is intricately related to the world around us, and no AI model can replace our human senses and our human curiosities. Discovering the right definition, the right mathematical concept: that is an endeavor far richer than any particular problem we may solve. Let us once again become explorers of this great unknown field of wonders instead of Game players wallowing in our ivory towers, afraid of our quicker and cleverer electronic competitors.

Unfortunately, at the moment such a renaissance of confidence in ourselves as humans and mathematicians seems unlikely, at least going on what I encountered at the Congress. It appears that everyone (I am of course no exception) is too caught up in their own particular Glass Bead Game, and, in this, Hesse once again presages with acumen. The novel's protagonist, Joseph Knecht (the surname roughly translates to "serf" from German), begins his journey as a young music student, but after several decades of intensive study is ultimately chosen as "Magister Ludi", the Master of the Game and the final authority over its rules. For a variety of reasons, however, Knecht's faith in the Game recedes as he grows into his role as champion player. By the examples and lessons of several of his closest life-companions, he rediscovers, in an age of petrification, that truly important scholarship is not simply a game, but rather a lifetime of human discovery and service. In addition, Knecht realizes that without these attributes, Castalia, with all

it represents, is in a precarious position. The question whether the rest of the world should continue to provide this scholarly realm with financial support is a significant concern. Knecht shares his thoughts and worries with the other head players of the Game, but they refuse to take his words into consideration. They are too absorbed in their single-minded game and too ignorant of the relationship between scholarship and world.

The service Knecht considers is not a simple and direct action (for example, volunteering to teach calculus at a prison), but rather a process of engaging with and contributing to the common culture of humanity. As mathematicians, we have many such services that we undertake, usually grudgingly. They include refereeing papers, writing textbooks, accepting and advising doctoral students, securing funding from national governments, teaching undergraduates, engaging with society, and apologizing for mathematics. We often discuss similar to-do lists with our colleagues, but how many of us view these cultural activities as simply obstacles to playing our own Glass Bead Game? How many professors fulfill their position in society in the etymological⁴ sense? An academic mathematician, in the fullest meaning, is one who can generate and transmit mathematical *ideas*, not one who is simply an adequate player of the mathematical game.

We have truly nothing to fear from AI models (that indeed may offer useful *tools* to the future mathematician), as long as we remember the human part of our profession. Mathematicians are not players of an abstract game, simply manipulating symbols in an aesthetically pleasing and intelligent way. Mathematicians are instead servants to our common human heritage, scholars in the truest sense: discoverers of the universe through use of the mind guided by both intuition and reason. Let us cast off the past century of game-playing and return to our role as natural scientists of the most rarefied sort. Mathematics is not a game or a sort of elegant chess to be "solved" by computer. Mathematics is the unexplored, and we may yet be excited and zealous in our exploration of

³Poincaré's flavor of intuitionism appeared to have been defeated by the influence of the axiomatic approach, championed by Hilbert and followed later by the Bourbaki group.

⁴The word "Professor" has the Latin root "profiteri", which means to "lay claim to" or "declare openly".

that unknown.

References

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