⇒ contact: mas10009@nyu.edu

⇒ Office Hours: 10am-11am Tuesdays & Wednesdays
in CIWW 905

↳ or by appointment in my office 907

⇒ recitation policies:

↳ quizzes will be administered in recitation, answers to be submitted on Brightspace by 11:59 pm same-day

Probability Basics

Recall that

event

$$P(X \in A)$$

probability

random variable

Q

Can someone tell me what it means for two events to be independent?

$$P(A \cap B) = P(A) \cdot P(B)$$

[equivalently, the fraction of event A that is also in event B = fraction of event B across all of $Z = \{\text{set of all outcomes}\}$]

Q **Example:**

Are employment status and glasses use independent?

	B employed	unemployed	total
A glasses	46	4	50
no glasses	14	26	40
total	60	30	90

$$P(A) = \frac{50}{90} = \frac{5}{9} \qquad P(A \cap B) = \frac{46}{90} \approx 0.5111$$

$$P(B) = \frac{60}{90} = 2/3 \Rightarrow P(A) \cdot P(B) \approx 0.37 \neq P(A \cap B)$$

They are dependent

Exponential/Radioactive Decay

dt : timestep/timescale

$N(t)$: number of atoms

↑ linked to spatial scale dV

δ : probability of an atom decaying over timestep dt

Q What defines a "good" dt?

timescale of individual atomic fluctuations $\ll dt \ll$ timescale at which $N(t)$ changes a lot

so we can assume randomness and independence (e.g. observation at dt doesn't predict fate at next dt)

so we can assume continuity of $N(t)$ because it varies slowly

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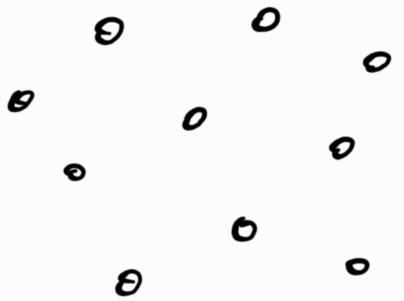


← consider all our N atoms

Q What is a <u>mean-field approximation</u> in this context?

⇒ we want to assume we have many atoms N, s.t. the <u>actual</u> # of atoms that decay over dt is relatively close to the <u>average</u> # expected

<u>Difference eqn</u> for the system:

$$N(t+dt) \underset{=}{\approx} N(t) \cdot (1-\delta)$$

If dt is small enough, we can assume continuity of N(t):

[Q] Example

suppose $\delta_1 = 0.002$ is the probability of your atom decaying over 1 week

① What is probability of survival over 1 week?

$$\eta_1 = 1-\delta_1 = 0.998$$

② What is probability of survival over 2 weeks?

$$\eta_2 = (1-\delta_1)\cdot(1-\delta_1) \approx 0.996$$

③ What about N weeks?

$$\eta_N = (1-\delta_1)^N$$

④ Let δ_N be probability of decaying over N weeks. How can we relate this to δ_1?

$$\delta_N = 1-\eta_N = 1-(1-\delta_1)^N$$

[N=52 → 1 year]

⑤ When does $\delta_N \simeq N\delta_1$?

How can we show this?

$$(1-\delta_1)^N = 1 - N\delta_1 + \frac{N(N-1)}{2!}\delta_1^2 - \frac{N(N-1)(N-2)}{3!}\delta_1^3 + \dots$$

↑
binomial series

if δ_1 is small, δ_1^2 is even smaller

$$\Rightarrow (1-\delta_1)^N \approx 1 - N\delta_1$$

$$\Rightarrow \delta_N = 1 - (1-\delta_1)^N = 1 - 1 + N\delta_1 = N\delta_1$$

□

Ordinary differential equation for the system:

$$\frac{d}{dt}N(t) = -\frac{\delta}{dt}N = -\tilde{\delta}N(t)$$

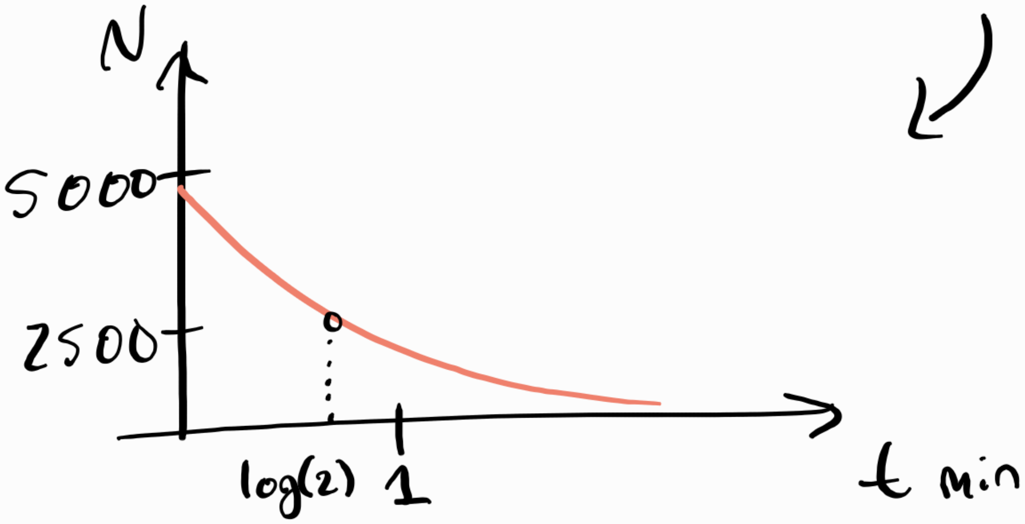
↗ decay rate

Q What is $\tilde{\delta}$ called? → decay rate

What are the units of $\tilde{\delta}$? 1/time

Q Can you give me an example of a solution to this? Is that the only solution?

$$\Rightarrow N(t) = N_0 \exp(-\tilde{\delta} t)$$



↙ suppose $N_0 = 5k$
and $\tilde{\delta} = 1$
$\Downarrow$
$\frac{\delta}{dt} \Rightarrow \delta$ pretty large

Q Do all N_0 choices result in solns that make sense?

Need $N_0 \geq 0$

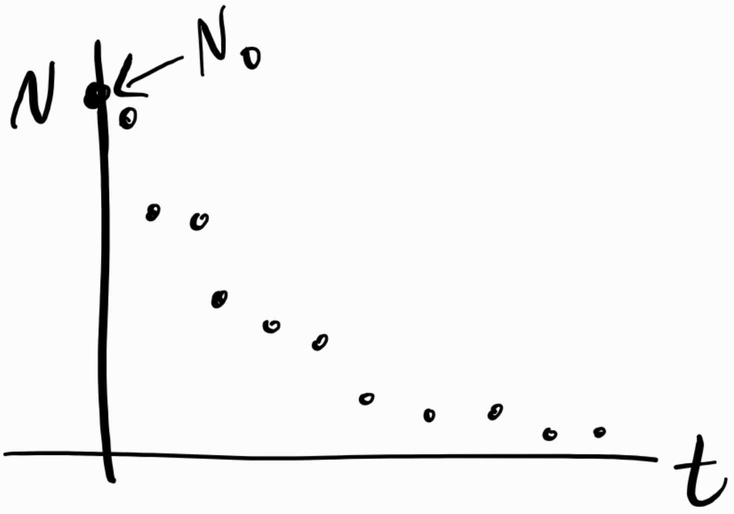
Q What happens if you start with no particles?

Notice that you will always have strictly non-negative-many particles with this model if $N_0 > 0$

Q Example

suppose we have a bunch of data for a radioactive atom pop.

①



② How do you know this is exponential?

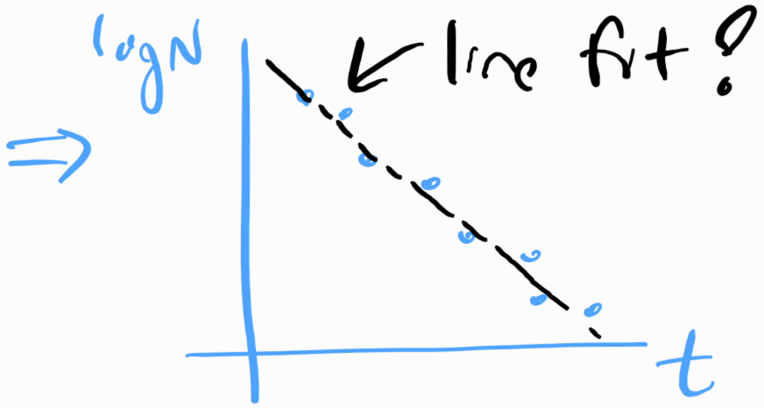
We expect $N(t) = N_0 \exp(-\delta t)$

Notice that

$$\log N = \log(N_0 \exp(-\delta t))$$

$$= \log N_0 + \log(\exp(-\delta t))$$

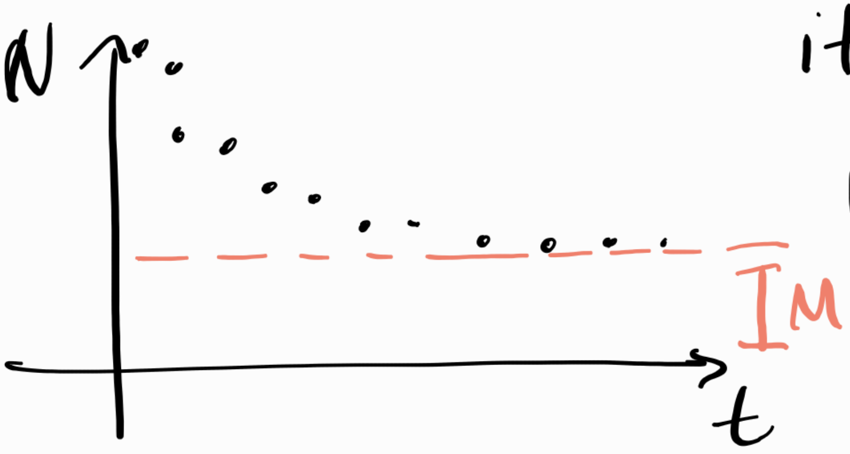
$$\underbrace{\log N = \log N_0 - \delta t}_{\text{this is linear!}}$$



③ How can you find the decay rate?

δ = Slope of the linear fit of data $\{(t, \log N(t))\}$

④ What if the data looks like this? Can you describe it with $\exp(-\delta t)$? How can you tell?



$$\lim_{t \to 0} N_0 \exp(-\delta t) = 0 \neq M$$

⑤ Why is there $M > 0$?
What could it represent?

Maybe some part of the population is not decaying ("inert"),

Or, part of the population decays at a different rate

halflife

⇒ e.g. Plutonium-238 87.7 yrs

Plutonium-239 ~24000 yrs

choice of timescale determines which you see

⑥ How can you "fix" this to find δ? subtract M

Generally, you might have multiple subpopulations decaying at different rates

i.e. $$N = \sum_{i=1}^{M} N_{0,i} \exp(-\delta_i t)$$

$\Rightarrow$ you cannot tell this just from data of $N(t)$, unless timescales are very different

[it is a poorly posed problem,

$\Rightarrow$ you do not always have a guaranteed soln, it is more likely to be impossible ?]

Quiz (5 minutes)

Suppose you draw a ticket from a box. The box has 6 tickets, each numbered 1, 2, or 3 and colored either blue or yellow. Can you find two independent events? If so, show why they are independent.

$$P(X=1) = \frac{1}{3}$$

$$P(X=\text{blue}) = \frac{1}{2}$$

$$P(X=1\,\&\,\text{blue}) = \frac{1}{6} = \frac{1}{3}\cdot\frac{1}{2} \checkmark$$