**Real Variables Fall 2011 (Young) HW 7 Due Oct 31**

1. Let $f : [a, b] \to \mathbb{R}$.

(a) Prove that if $f$ is Lipschitz, then it is absolutely continuous. Is the converse true?

(b) Prove that if $f$ is absolutely continuous, and $E \subset [a, b]$ is a set of measure zero, then $f(E)$ has measure zero.

2. Let $g : [a, b] \to \mathbb{R}$ be integrable, and define

$$f(x) = \int_a^x g \ .$$

Prove that $f$ has bounded variation, hence $f'$ exists a.e.

3. (a) Let $\varphi : [a, b] \to \mathbb{R}$ be strictly convex. Prove that $\varphi'_\pm$ (the left and right derivatives of $\varphi$) are strictly increasing.

(b) Let $g$ be a nonnegative measurable function on $[0, 1]$. Prove that

$$\log \int g \geq \int \log g$$

whenever the right side is defined, and equality holds if and only if $g \equiv$ constant.