1. Compute $\delta_{ij}\delta_{jk}\delta_{ki}$

   Solution:

   $\delta_{ij}\delta_{jk}\delta_{ki} = \delta_{ik}\delta_{ki} = \delta_{ii} = 3$

2. Use suffix notation to show

$$(\vec{a} \times \vec{b}) \times \vec{c} + (\vec{b}.\vec{c})\vec{a} = \vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a}.\vec{b})\vec{c}$$

   Solution:

$$\begin{aligned}
&[(\vec{a} \times \vec{b}) \times \vec{c} + (\vec{b}.\vec{c})\vec{a}]_i \\
=&\epsilon_{ijk}(\vec{a} \times \vec{b})_j c_k + b_j c_j a_i \\
=&\epsilon_{ijk}\epsilon_{jlm} a_l b_m c_k + b_j c_j a_i \\
=&(\delta_{kl}\delta_{im} - \delta_{km}\delta_{il}) a_l b_m c_k + b_j c_j a_i \\
=&a_k b_i c_k - a_i b_k c_k + b_j c_j a_i \\
=&a_k c_k b_i
\end{aligned}$$

$$\begin{aligned}
&[\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a}.\vec{b})\vec{c}]_i \\
=&\epsilon_{ijk} a_j (\vec{b} \times \vec{c})_k + a_j b_j c_i \\
=&\epsilon_{ijk} a_j \epsilon_{klm} b_l c_m + a_j b_j c_i \\
=&\epsilon_{ijk}\epsilon_{klm} a_j b_l c_m + a_j b_j c_i \\
=&(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) a_j b_l c_m + a_j b_j c_i \\
=&a_j b_i c_j - a_j b_j c_i + a_j b_j c_i \\
=&a_j c_j b_i
\end{aligned}$$

   So we see the left side equals the right side.

3. Use suffix notation to show $\vec{\nabla} \times (f\vec{\nabla} f) = 0$

   Solution:

$$\begin{aligned}
&[\vec{\nabla} \times (f\vec{\nabla} f)]_i \\
=&\epsilon_{ijk}\frac{\partial}{\partial x_j}(f\frac{\partial f}{\partial x_k}) \\
=&\epsilon_{ijk}\frac{\partial f}{\partial x_j}\frac{\partial f}{\partial x_k} + \epsilon_{ijk} f\frac{\partial}{\partial x_j}\frac{\partial f}{\partial x_k} \\
=&[\vec{\nabla} f \times \vec{\nabla} f]_i + f[\vec{\nabla} \times (\vec{\nabla} f)]_i \\
=&0
\end{aligned}$$

4. Use suffix notation to show $\vec{\nabla}.(\nabla^2\vec{u}) = \nabla^2(\vec{\nabla}.\vec{u})$

   Solution:

$$\begin{aligned}
\vec{\nabla}.(\nabla^2\vec{u}) =&\frac{\partial}{\partial x_i}(\nabla^2 u_i) \\
=&\frac{\partial}{\partial x_i}\frac{\partial^2 u_i}{\partial x_j \partial x_j} \\
=&\frac{\partial^3 u_i}{\partial x_i \partial x_j \partial x_j} \\
=&\frac{\partial^2}{\partial x_j \partial x_j}\frac{\partial u_i}{\partial x_i} \\
=&\nabla^2(\vec{\nabla}.\vec{u})
\end{aligned}$$

5. An equation is given in terms of the suffix notation as

$$u_t\frac{\partial u_i}{\partial x_t} = \frac{1}{2}\frac{\partial(u_k u_k)}{\partial x_i} - \epsilon_{ijk}\epsilon_{klm} u_j \frac{\partial u_m}{\partial x_l}$$

   (i). Write this equation in vector form.

   Solution:

$$\vec{u}.\vec{\nabla}\vec{u} = \frac{1}{2}\vec{\nabla}(\vec{u}.\vec{u}) - \vec{u} \times (\vec{\nabla}\vec{u})$$

(ii). Prove this equation.

Solution:

$$\begin{aligned}
&\frac{1}{2}\frac{\partial(u_k u_k)}{\partial x_i} - \epsilon_{ijk}\epsilon_{klm}u_j\frac{\partial u_m}{\partial x_l} \\
=&u_k\frac{\partial u_k}{\partial x_i} - (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})u_j\frac{\partial u_m}{\partial x_l} \\
=&u_k\frac{\partial u_k}{\partial x_i} - u_j\frac{\partial u_j}{\partial x_i} + u_j\frac{\partial u_i}{\partial x_j} \\
=&u_j\frac{\partial u_i}{\partial x_j} \\
=&u_t\frac{\partial u_i}{\partial x_t}
\end{aligned}$$