1. Translate the suffix notation $\delta_{ij}c_j + \epsilon_{kji}a_k b_j = d_l e_m c_i b_l c_m$ into ordinary vector equation.

   Solution: This suffix notation can be rewritten as

$$c_i + \epsilon_{ikj}a_k b_j = b_l d_l c_m e_m c_i$$

   So it stands for the expression

$$\vec{c} + \vec{a} \times \vec{b} = (\vec{b}.\vec{d})(\vec{c}.\vec{e})\vec{c}$$

2. Use suffix notation to show that the $n \times n$ identity matrix commutes with any $n \times n$ matrix with respect to matrix multiplication.

   Solution:

$$\delta_{ij}A_{jk} = A_{ik} = A_{ij}\delta_{jk}$$

3. Compute $\epsilon_{ijk}\epsilon_{ijk}$

   Solution: We know $\epsilon_{ijk}^2 = 0$ if any of i, j, k are the same, and $\epsilon_{ijk}^2 = 1$ if i, j, k are distinct, so

$$\epsilon_{ijk}\epsilon_{ijk} = \epsilon_{123}^2 + \epsilon_{231}^2 + \epsilon_{312}^2 + \epsilon_{132}^2 + \epsilon_{213}^2 + \epsilon_{321}^2 = 6$$

4. Use Suffix notation to show $\vec{a}.(\vec{b} \times \vec{c}) = -\vec{c}.(\vec{b} \times \vec{a})$

   Solution:

$$\begin{aligned} a_i(\vec{b} \times \vec{c})_i &= a_i \epsilon_{ijk} b_j c_k \\ &= -c_k \epsilon_{kji} b_k a_i \\ &= -c_k(\vec{b} \times \vec{a})_k \end{aligned}$$

5. Using suffix notation to find an alternative expression for $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$ which doesn't involve cross product.

   Solution:

$$\begin{aligned}
(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) &= \epsilon_{ijk} a_j b_k \epsilon_{ilm} c_l d_m \\
&= \epsilon_{ijk} \epsilon_{ilm} a_j b_k c_l d_m \\
&= (\sigma_{jl}\sigma_{km} - \sigma_{jm}\sigma_{kl}) a_j b_k c_l d_m \\
&= a_j b_k c_j d_k - a_j b_k c_k d_j \\
&= (\vec{a}.\vec{c})(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})(\vec{b}.\vec{c})
\end{aligned}$$

6. If A, B are two $n \times n$ matrices, use suffix notation to prove $(AB)^T = B^T A^T$, where ^{T} means transpose

   Solution:

$$\begin{aligned}
(AB)^T_{ij} &= (AB)_{ji} \\
&= A_{jk} B_{ki} \\
&= B_{ki} A_{jk} \\
&= B^T_{ik} A^T_{kj} \\
&= (B^T A^T)_{ij}
\end{aligned}$$