1. $\vec{F}(x,y,z) = (x,y,z)$. Prove there doesn't exist a vector field $\vec{G}$ such that $\vec{F} = \vec{\nabla} \times \vec{G}$.

2. Compute the volume of the solid $E = \{(x,y,z) \in \mathbb{R}^3 : x^2 + z^2 \leq 1, y^2 + z^2 \leq 1\}$

3. $f(x,y,z)$ is a scalar function and $\vec{F}(x,y,z)$ is a vector field. Prove
$$\vec{\nabla}.(f\vec{F}) = f(\vec{\nabla}.\vec{F}) + (\vec{\nabla}f).\vec{F}$$

4. Let E be the unit cube $E = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$. Let S be the boundary of E with outward orientation. If $\vec{F}(x,y,z) = (2xy, 3ye^z, x \sin z)$, compute $\oiint_S \vec{F}.\,d\vec{S}$

5. $\vec{F}(x,y,z)$ is a vector field defined on $\mathbb{R}^3$. $0 < r < R$ are constants. Let S_R (S_r) denote the sphere of radius R (r) centered at origin. E is the region bounded by these two spheres. Prove
$$\iiint_E div\vec{F}\,dV = \oiint_{S_R} \vec{F}.\,d\vec{S} - \oiint_{S_r} \vec{F}.\,d\vec{S}$$