1. $\{\vec{v}_1, ..., \vec{v}_n\}$ are n linearly independent vectors in $\mathbb{R}^n$. If $\vec{v}.\vec{v}_i = 0$ for any $i \in \{1, 2, ..., n\}$, prove $\vec{v} = \vec{0}$

2. $\vec{S}$ is an oriented surface with boundary a closed curve C with orientation compatible with that of $\vec{S}$. $f(x, y, z)$ is a smooth scalar function defined in a region containing $\vec{S}$.

   (i). Prove that for any unit vector $\hat{u}$,

   $$\hat{u}.(\iint_S -\vec{\nabla} f \times d\vec{S}) = \hat{u}.(\oint_C f \, d\vec{r})$$

   (Hint: Apply Stokes' Theorem to $f\hat{u}$)

   (ii). Prove $\iint_S -\vec{\nabla} f \times d\vec{S} = \oint_C f \, d\vec{r}$

3. f, g, h are single-variable smooth functions $\mathbb{R} \longrightarrow \mathbb{R}$. Show that the vector field $\vec{F}(x, y, z) = (f(x), g(y), h(z))$ is conservative.

4. Use Stokes' Theorem to evaluate $\iint_S \vec{\nabla} \times \vec{F}.\, d\vec{S}$ where $\vec{F} = (y^2 z, zx, x^2 y^2)$, and $\vec{S}$ is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 1$, oriented upward.

5. S is the cylinder $x^2 + y^2 = 1, 0 \leq z \leq 1$. The two boundary circles C_1 and C_2 are oriented counterclockwise seen from above. If $\vec{F}$ is a vector field on $\mathbb{R}^3$ such that $\iint_S \vec{\nabla} \times \vec{F}.\, d\vec{S} = 0$, prove

   $$\int_{C_1} \vec{F}.\, d\vec{r} = \int_{C_2} \vec{F}.\, d\vec{r}$$