1. $\vec{F}(x, y, z) = (y, z, x)$ is a vector field. $\vec{S}$ is the unit sphere $x^2 + y^2 + z^2 = 1$ with outward orientation. Compute

$$\iint_S \vec{F} \times d\vec{S}$$

2. $\vec{F}(x, y, z) = x + y + z$ is a scalar function. C is the oriented intersection curve of the paraboloid $z = x^2 + y^2$ and the plane $x + y = 0$ from $(0, 0, 0)$ to $(1, -1, 2)$. Compute

$$\int_C f \, d\vec{r}$$

3. If $f(x, y, z)$ is a scalar function and $\vec{F}(x, y, z)$ is a vector field, prove

$$\vec{\nabla} \times (f\vec{F}) = f(\vec{\nabla} \times \vec{F}) + \vec{\nabla} f \times \vec{F}$$

4. If $f(x, y, z)$ and $g(x, y, z)$ are scalar functions, $\vec{F} = g\vec{\nabla} f$, show that

$$\vec{F}.Curl(\vec{F}) = 0$$

5. Show that $\vec{F}(x, y, z) = (y, z, x)$ is not a conservative vector field.