

1. Compute $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (-y, x)$, and C is the closed path on the circle $x^2 + y^2 = R^2$ along counterclockwise direction.
2. If C is a smooth curve in \mathbb{R}^2 parameterized by $\vec{r}(t), a \leq t \leq b$, and \vec{v} is a constant vector, show that

$$\int_C \vec{v} \cdot d\vec{r} = \vec{v} \cdot [\vec{r}(b) - \vec{r}(a)]$$

3. Find a potential function of the vector field

$$\vec{F}(x, y) = \langle xy^2, x^2y \rangle$$

4. (i). Show that the line integral is independent of path

$$\int_C 2xe^{-y} dx + (2y - x^2e^{-y}) dy$$

C is any path from $(1, 0)$ to $(2, 0)$.

- (ii). Evaluate the above integral

5. $\vec{F}(x, y) = (P(x, y), Q(x, y))$ is a conservative vector field defined on \mathbb{R}^2 such that P and Q are smooth functions, and $\vec{F} \neq (0, 0)$ for all points. If $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a smooth function, prove $g\vec{F}$ is conservative if and only if ∇g is parallel to \vec{F} everywhere.