1. $F : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ and $G : \mathbb{R}^m \longrightarrow \mathbb{R}^k$ are differentiable maps. Prove the chain rule:
$$D(G \circ F)(x) = DG(F(x))DF(x)$$

2. (1). $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a differentiable function such that $f'(x) \neq 0$ for any $x \in \mathbb{R}$. Prove f is a one-to-one function.

   (ii). Prove $p : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $p(x) = x^5 + x^3 + x + 1$ is a bijective function.

   (iii). Compute $(p^{-1})'(1)$

3. Verify that the function $F(x, y) = (x^2 + y^2, x - y^3)$ is locally invertible at $(0, 1)$, and compute $DF^{-1}(1, -1)$

4. $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a continuously differentiable function. If there is a sequence $\{x_n\} \in \mathbb{R}^n$ such that $\lim_{x\to\infty} x_n = x_0$, $x_0 \neq x_n$ for any $n \in \mathbb{N}$, and $f(x_n) = a \in \mathbb{R}^n$ for any $n \in \mathbb{N}$, prove $DF(x_0)$ is not invertible.

5. $F(x, y, z) = (x + y + z, x^2 + y^2 + z^2)$. Prove there exist differentiable $g(x)$ and $h(x)$ defined in some neighbourhood of $x = 0$ such that $g(0) = 1, h(0) = 2$ and on this neighbourhood $F(x, g(x), h(x)) = (3, 5)$. Compute $g'(0)$ and $h'(0)$.

6. A surface is defined by the equation $xy + y^2z + z^3x = 3$. Find the equation of the tangent plane for the surface at $(1, 1, 1)$.