1. If $T_{ij}$ represents a 2-tensor, prove $T_{ii}$ is independent of coordinates.

2. If $T_i$ and $S_j$ represent 1-tensors, prove $T_i S_j$ represents a 2-tensor.

3. If $T_{ijkl}$ represents a tensor of rank 4, prove $T_{ijjl}$ represents a tensor of rank 2.

4. Construct an isotropic 5-tensor on $\mathbb{R}^3$ and prove it is isotropic.

5. (i). If $x_i$ represents a 1-tensor, and $A = (a_{ij})$ denotes the matrix of change of basis, prove that $\frac{\partial x'_i}{\partial x_j} = a_{ij}$ and $\frac{\partial x_i}{\partial x'^j} = a_{ji}$

   (ii). If $f$ is a smooth function, prove $\frac{\partial f}{\partial x_i}$ represents a 1-tensor.

   (iii). If $u_i$ represents a 1-tensor, prove $\frac{\partial u_i}{\partial x_j}$ represents a 2-tensor.

6. $\sigma \in S_n$. Define the $n \times n$ matrix $A = (\delta_{i\sigma^{-1}(j)})$.

   (i). If $n = 3$, $\sigma(1) = 3, \sigma(2) = 1, \sigma(3) = 2$, write out $A$ explicitly.

   (ii). Prove $A \in O_n(\mathbb{R})$.

   (iii). Prove $A \in SO_n(\mathbb{R})$ if and only if $\text{sgn}(\sigma) = +1$. 