1. If T_{ij} represents a 2-tensor, prove T_{ii} is independent of coordinates.
2. If T_i and S_j represent 1-tensors, prove T_iS_j represents a 2-tensor.
3. If T_{ijkl} represents a tensor of rank 4, prove T_{ijjl} represents a tensor of rank 2.
4. Construct an isotropic 5-tensor on $\mathbb{R}^3$ and prove it is isotropic.
5. (i). If x_i represents a 1-tensor, and $A = (a_{ij})$ denotes the matrix of change of basis, prove that
$$\frac{\partial x_i'}{\partial x_j} = a_{ij} \text{ and } \frac{\partial x_i}{\partial x_j'} = a_{ji}$$
(ii). If f is a smooth function, prove $\frac{\partial f}{\partial x_i}$ represents a 1-tensor.
(iii). If u_i represents a 1-tensor, prove $\frac{\partial u_i}{\partial x_j}$ represents a 2-tensor.
6. $\sigma \in S_n$. Define the $n \times n$ matrix $A = (\delta_{i\sigma^{-1}(j)})$.
(i). If $n = 3$, $\sigma(1) = 3, \sigma(2) = 1, \sigma(3) = 2$, write out A explicitly.
(ii). Prove $A \in O_n(\mathbb{R})$.
(iii). Prove $A \in SO_n(\mathbb{R})$ if and only if $sgn(\sigma) = +1$.