Homework II First-Half

Due in class Monday July 16 2017

1. The utility function is \( u(x, y) = Ax^ay^b \). The unit price of \( x \) is \( p \) dollars and the unit price for \( y \) is \( q \) dollars. Find \( x \) and \( y \) to maximize the utility subject to \( m \) dollars budget constraint.

2. \( u(x, y) \) is a utility function, and \( g(x, y) \) is the budget function. When the budget is 200, it is known the maximal utility is 125. If we know at this point the Lagrange multiplier is \( \lambda = 3 \), estimate the maximal utility when the budget is 198

3. Minimize \( f(x, y, z) = x^2 + y^2 + z^2 \) subject to the constraint \( x+y+z = 1 \).

4. Minimize \( f(x, y, z) = x^2 + y^2 + z^2 \) subject to the constraints \( x+y+z = 1 \) and \( x - y + z = 3 \)

5. There is a string of length \( L \). Mickey cuts the string into two segments, of length \( x \) and \( y \) respectively. He uses the length \( x \) segment to make a square and the length \( y \) segment to make a circle. Find \((x, y)\) that minimizes the total area enclosed by the square and the circle.

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